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Low Energy $\overline{K}$-Nucleon Scattering.

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Summary. — The *s*-wave low energy scattering of $\overline{K}$-mesons on nucleons is studied using a similar method as that applied, in a previous work, for the K-N interaction. Disregarding virtual barion-antibarion pair effects, K virtual state contributions and barion recoil, the integral equations for the two isobaric spin state amplitudes are exactly solved, and analytical expressions for the phase shifts are given. Results are rather independent on the cut-off energy and indicate repulsive potential for both isobaric spin eigenstates. Numerical results are presented for the values of the coupling constants suggested in the previous work.

In order to study the *s*-wave low energy scattering of $\overline{K}$-mesons (K^- and $\overline{\theta}^0$) on nucleons, we shall use a method similar to that applied in a previous work on the K-N interaction [1]. The same interaction Hamiltonian for the

[1] D. AMATI and B. VITALE: *Nuovo Cimento* (in press). We refer to that paper (in the following called A) for the assumed properties of particles and the meaning of symbols.

N-K-Λ^0-Σ system is considered, i.e.

$$(1)\qquad H' = g^{\Lambda}[\overline{\psi}_N \varphi_\Lambda A_K + A_K^+ \overline{\varphi}_\Lambda \psi_N] + g_\Sigma[\overline{\psi}_N \boldsymbol{\tau}\cdot\boldsymbol{\varphi}_\Sigma A_K + A_K^+ \overline{\boldsymbol{\varphi}}_\Sigma \cdot \boldsymbol{\tau}\psi_K] + \\ + \delta m_K \int \overline{\psi}_N \psi_N \, d\tau + \delta m_\Lambda \int \overline{\varphi}_\Lambda \varphi_\Lambda \, d\tau + \delta m_\Sigma \int \overline{\boldsymbol{\varphi}}_\Sigma \cdot \boldsymbol{\varphi}_\Sigma \, d\tau \,,$$

where

$$A_K = \sum_k \frac{v(k)}{(2\omega_k)^{\frac{1}{2}}} (a_k \exp[i\boldsymbol{k}\cdot\boldsymbol{r}] + b_k^+ \exp[-i\boldsymbol{k}\cdot\boldsymbol{r}]) \,.$$

As in *A* we disregard contributions of pions (absorption processes are not considered), the recoil of the barion and virtual barion-antibarion pair effects. Also we shall not take into account contributions to the $\overline{K}$-N interaction coming from the virtual K creation and annihilation (other than the obvious self-energy of nucleons). These assumptions bring us to an exactly solvable field theoretical problem, similar to that studied by T. D. LEE ([2]).

1. – We can now express the « dressed » (physical) states of Λ and Σ particles as a linear combination of bare states, and $\overline{K}$-meson plus nucleon:

$$(2)\qquad \begin{cases} |\underline{\Lambda}\rangle = B[|\Lambda\rangle + \sum_k b_k^+ \xi(k) |N\rangle] \\ |\underline{\Sigma}_e\rangle = C[|\Sigma\rangle + \sum_k b_k^+ \zeta(k) |N\rangle] \,. \end{cases}$$

If m_Λ and m_Σ are the « observed » masses, then

$$(3)\qquad H|\underline{\Lambda}\rangle = m_\Lambda |\underline{\Lambda}\rangle \,; \qquad H|\underline{\Sigma}_e\rangle = m_\Sigma |\underline{\Sigma}\rangle$$

using (1), (2) and (3) we obtain

$$(4)\qquad \begin{cases} \xi(k) = -\dfrac{g_\Lambda v(k)}{\sqrt{2\omega_k}(\omega_k + m_N - m_\Lambda)}; \qquad \zeta^e(k) = -\dfrac{g_\Sigma v(k)\tau^e}{\sqrt{2\omega_k}(\omega_k + m_N - m_\Sigma)} \,, \\ \delta m_\Lambda = 2g_\Lambda^2 \sum_k \dfrac{v^2(k)}{(2\omega_k)(\omega_k + m_N - m_\Lambda)}; \qquad \delta m_\Sigma = 2g_\Sigma^2 \sum_k \dfrac{v^2(k)}{(2\omega_k)(\omega_k + m_N - m_\Sigma)} \,, \end{cases}$$

and from the normalization of (2)

$$(6)\qquad \begin{cases} B = [1 + \sum_k \xi^2(k)]^{-\frac{1}{2}} \,, \\ C = [1 + \sum_{k,e} (\zeta^e(k))^2]^{-\frac{1}{2}} \,. \end{cases}$$

([2]) T. D. LEE: *Phys. Rev.*, **95**, 1329 (1954).

2. – The physical K-N states can be expressed in terms of single particle states. Separating the two total isobaric spin eigenstates, we have:

for $T = 0$

$$|\underline{\overline{\mathrm{K}}+\mathrm{N}}\rangle_0 = c\,|\underline{\Lambda}\rangle + \sum_k b_k^+ \chi_0(k)\,|\mathrm{N}\rangle \tag{7a}$$

and for $T = 1$

$$|\underline{\overline{\mathrm{K}}+\mathrm{N}}\rangle_1^e = d\,|\underline{\Sigma}\rangle_e + \sum_k b_k^+ \chi_1^e(k)\,|\mathrm{N}\rangle\,. \tag{7b}$$

From the condition

$$H\,|\underline{\overline{\mathrm{K}}+\mathrm{N}}\rangle = (\omega_p + m_\mathrm{N})\,|\underline{\overline{\mathrm{K}}+\mathrm{N}}\rangle\,, \tag{8}$$

where ω_p is the total energy of the incident meson, we then obtain for the $T = 0$ and $T = 1$ scattering amplitudes the following integral equations:

$$(\omega_k - \omega_p)\chi_0(k) = \int K_0(k, k')\chi_0(k')\,\mathrm{d}k'\,, \tag{9a}$$

$$\chi_1^e = \tau^e \chi_1\,, \qquad (\omega_k - \omega_p)\chi_1(k) = \int K_1(k, k')\chi_1(k')\,\mathrm{d}k'\,, \tag{9b}$$

where

$$K_0(k, k') = 2G_\Lambda^2(m_\Lambda - m_\mathrm{N} - \omega_p)(4\omega_k\omega_{k'})^{-\frac{1}{2}}k'^2 \cdot \\ \cdot[2\pi^2(m_\Lambda - m_\mathrm{N} - \omega_k)(m_\Lambda - m_\mathrm{N} - \omega_{k'})]^{-2}v(k)v(k') \tag{10}$$

and a similar expression for $K_1(k, k')$ with m_Λ and G_Λ replaced respectively by m_Σ and G_Σ. G_Λ and G_Σ are the renormalized and rationalized coupling constants given by

$$G_\Lambda = Bg_\Lambda\,, \qquad G_\Sigma = Cg_\Sigma\,. \tag{11}$$

Owing to the separability of the kernels, the integral equations (9) can be exactly solved, the result being

$$\chi_n(k) = \delta(k - p) + K_1(k, p)(\omega_k - \omega_p)^{-1}\left[1 - \mathcal{P}\int \frac{K_k(k', k')\,\mathrm{d}k'}{\omega_{k'} - \omega_p}\right]^{-1}, \tag{12}$$

where $\mathcal{P}$ stands for the principal value of the integral. Performing the integration we find for the phase shifts the following expression:

$$\mathrm{tg}\,\delta_0(p) = -\frac{G_\Lambda^2}{2\pi}\frac{p}{(\omega_p - m_\Lambda + m_\mathrm{N})}\left[1 + \frac{2G_\Lambda^2}{\pi^2}\cdot\frac{m_\Lambda^2(\omega_p - m_\Lambda + m_\mathrm{N})}{(\omega_p + m_\mathrm{K})(m_\mathrm{K} - m_\Lambda + m_\mathrm{N})^2}\cdot\right. \\ \left.\cdot\left\{\frac{t}{(b^2 + t^2)^2}\log\frac{a - t}{a + t} + \frac{b^2 - t^2}{b(b^2 + t^2)^2}\,\mathrm{arctg}\,\frac{a}{b} + \frac{a}{(b^2 + t^2)(b^2 + a^2)}\right\}\right]^{-1}, \tag{13}$$

where

$$a = \left(\frac{\eta - m_K}{\eta + m_K}\right)^{\frac{1}{2}}, \qquad b = \left(\frac{m_K - (m_\Lambda - m_N)}{m_K + (m_\Lambda - m_N)}\right)^{\frac{1}{2}}, \qquad t = \left(\frac{\omega_p - m_K}{\omega_p + m_K}\right)^{\frac{1}{2}}.$$

and η is the cut-off energy.

A similar formula is obtained for δ_1 with m_Λ and G_Λ replaced respectively by m_Σ and G_Σ.

3. – As was to be expected from the beginning, in the expression for the $T = 0$ phase shift only the renormalized coupling constant for the K-N-Λ interaction is involved; for the $T = 1$ only the K-N-Σ coupling constant. This was not the case for the results obtained in *A* for the K-N phase shifts, there the scattering cross-section for both isobaric spin states were dependent on the two coupling constants.

From the expressions for the phase shifts, we see easily that these are weakly dependent on the cut-off energy η. We see also that for the energies considered (say lower than 200 MeV) the phase shifts are negative for all values of the coupling constant; this indicates a repulsive $\overline{\text{K}}$-nucleon potential for both values of T.

In *A* some values of G_Λ and G_Σ were considered so as to fit experimental results on K^+ nucleus scattering. For these values of the coupling constants, we give in Table I numerical results computed for 30 MeV and 100 MeV kinetic energy of the incident mesons. The tabulated data correspond to $T = 0$ if the coupling constant considered is G_Λ, and to $T = 1$ if it is G_Σ, disregarding the mass difference of Λ and Σ hyperons. We used a cut-off energy of 3 000 MeV.

TABLE I.

$G^2/4\pi$	30 MeV		100 MeV	
	δ	σ in mb	δ	σ in mb
0.3	— 12°	7	— 21°	6
0.4	— 15°	10	— 26°	8
0.8	— 21°	19	— 37°	16

Although experimental information on K^- scattering in hydrogen and in nuclear emulsion is rapidly growing (as well as experiences with long lived θ^0 particles), present data are yet very scarse. ALVAREZ *et al.* [3] on the basis of five visible events of K^- scattering in hydrogen (three elastic and two charge

[3] L. W. ALVAREZ, H. BRADNER, P. FALK-VAIRANT, J. D. GOW, A. H. ROSENFELD, F. T. SOLMITZ and R. D. TRIPP: UCRL report No. 3583 (unpublished).

exchange) predict a scattering cross-section of (45 ± 30) mb at a K^- energy of about 30 MeV. Owing to the few experimental data accumulated so far, no realistic conclusions can be drawn from the comparison with our theoretical results. It must also be remembered, in order to carry on this comparison, that the shadow scattering of absorption processes must be added to the results here obtained. However, this contribution would be in general concentrated in rather small angles so that it might escape experimental observation.

RIASSUNTO

Si studia la diffusione $\overline{K}$-Nucleone in onda *s* a basse energie, con un metodo simile a quello usato, in un lavoro precedente, per l'interazione K-N. Trascurando gli effetti di coppie virtuali di barioni e antibarioni, i contributi dei K virtuali ed il rinculo del barione, si ottengono equationi integrali per le ampiezze di diffusione nei due stati di spin isobarico. Queste equazioni si risolvono esattamente e si ottengono espressioni analitiche per gli sfasamenti. I risultati sono poco sensibili al valore di taglio di energia e indicano potenziali repulsivi per entrambi gli stati di spin isobarico. Si danno infine le sezioni d'urto per alcuni valori dell'energia dei $\overline{K}$ e per i valori delle costanti d'accoppiamento indicate dal precedente lavoro.

Invariance Properties of Fermi Interactions.

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Summary. — The effect of vanishing observable neutrino mass on the conditions that Fermi interactions should be invariant under P, T, and C is investigated. A minor correction is given to the formula for $\beta - \nu$ angular correlation given by Lee and Yang. Results are obtained which reduce the labour of extending existing calculations to interactions which are not invariant under P, T or C. Some obscurities in the existing literature on β-spectrum shape correction factors are clarified. It is shown that in μ-meson decay, to lowest order, there can be no interferences between the group of interactions (V, A) and the group (S, T, P). An alternative to Salam's neutrino gauge hypothesis is indicated. The foundations of the method used in the paper are discussed. A general theorem on weak interactions which are invariant under P but not under T or C, together with two corollaries, is stated in an appendix, and the proof is outlined.

1. – Introduction.

Recent experiments by Wu *et al.* [1] and by Garwin *et al.* [2] have confirmed the suggestion made by Lee and Yang [3] that parity may not be conserved in weak interactions. Lee, Oehme and Yang [4] have discussed some consequences of the Schwinger-Lüders-Pauli theorem [5], and in part-

[1] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson: *Phys. Rev.*, **105**, 1413 (1957).

[2] R. L. Garvin, L. M. Lederman and M. Weinrich: *Phys. Rev.*, **105**, 1415 (1957).

[3] T. D. Lee and C. N. Yang: *Phys. Rev.*, **104**, 254 (1956).

[4] T. D. Lee, R. Oehme and C. N. Yang: *Phys. Rev.*, **106**, 340 (1957).

[5] W. Pauli: article in *Niels Bohr and the Development of Physics* (London, 1955).

icular have pointed out that parity non-conserving interactions must be non-invariant under at least one of particle-antiparticle conjugation (C) and time reversal (T) as well as space inversion (P). This paper primarily investigates the effect of the assumption of zero mass for the neutrino on the conditions for the invariance of Fermi interactions under these transformations, and also derives further consequences of the vanishing neutrino mass which greatly reduce the labour of calculations with the theory.

It is well-known that the theory is unaltered if all the coupling constants of the Fermi interactions are multiplied by the same phase factor. This phase factor could be absorbed into one of the fields in the interaction by means of a unitary transformation, or alternatively it is possible to choose the undetermined phases in the transformations of the fields under C, P and T so that the invariance properties of the interaction are unaltered. This paper is based on the fact that if the mass-less neutrino field is subdivided into two dynamically independent two-component fields,

$$\psi_\nu = \psi_1 + \psi_2 , \tag{1}$$

defined by the equations

$$\psi_1 = \tfrac{1}{2}(1 + \gamma_5)\psi_\nu , \qquad \psi_2 = \tfrac{1}{2}(1 - \gamma_5)\psi_\nu , \tag{2}$$

then it is possible by a unitary transformation to absorb different phase factors into ψ_1 and ψ_2. (Hermitian γ-matrices are used throughout). The unitary transformations considered are those which bring about the substitution

$$\psi_\nu \to \exp\left[-\tfrac{1}{2}i\theta(1 + \gamma_5) - \tfrac{1}{2}i\varphi(1 - \gamma_5)\right]\psi_\nu . \tag{3}$$

The corresponding ambiguities in the definition of the transformation properties under P, T and C for a mass-less spinor field have been noted by SHAW ([6]); they were rediscovered recently by WERLE and ONEDA ([7]).

The importance of (3) is that when the exponential factor is combined with the coupling constants instead of with the fields, the relative magnitudes as well as the phases of the coupling constants are changed for « even » and « odd » terms. A well-known example is $\theta = 0$, $\varphi = \pi$, for which (3) becomes

$$\psi_\nu \to \gamma_5\psi_\nu ,$$

and a purely « even » interaction is transformed into a purely « odd » one.

([6]) R. SHAW: *Ph. D. Dissertation*, Cambridge University (1955) (unpublished).

([7]) J. WERLE and S. ONEDA: private communication (1957).

2. – Nuclear β decay. Invariance conditions.

The notations used are those of the Appendix to reference [3]. Sufficient conditions for invariance of H_{int} under P, C and T are:

(i) P invariance: all $C_j' = 0$;
(ii) T invariance: all C_j, C_j' real;
(iii) C invariance: all C_j real, all C_j' pure imaginary.

These conditions, however, are not necessary.

Consider first the odd and even interaction terms of a given tensor character j. These can be written

$$(4) \qquad \psi_p^\dagger \gamma_4 \mathcal{O}_{j\mu} \psi_n \psi_e^\dagger \gamma_4 \mathcal{O}_{j\mu} (C_j + C_j' \gamma_5) \psi_\nu =$$
$$= \psi_p^\dagger \gamma_4 \mathcal{O}_{j\mu} \psi_n \psi_e^\dagger \gamma_4 \mathcal{O}_{j\mu} (B_j + B_j' \gamma_5) \exp[\tfrac{1}{2} i\theta(1+\gamma_5) + \tfrac{1}{2} i\varphi(1-\gamma_5)] \psi_\nu .$$

The exponential term in (4) could be absorbed into ψ_ν by the unitary transformation (3), so that the coupling constants B_j, B_j' are on exactly the same footing as C_j, C_j'. The required necessary and sufficient conditions for invariance are that it is possible to choose θ and φ so that the B_j and B_j' satisfy the conditions (i), (ii) or (iii). These give

(*a*) P invariance:	$C_j C_k'^* + C_j' C_k^* = 0$,		all j and k;
(*b*) T invariance:	$\mathrm{Im}\,(C_j C_k^* + C_j' C_k'^*) = 0$,		
and	$\mathrm{Im}\,(C_j C_k'^* + C_j' C_k^*) = 0$,		all j and k;
(*c*) C invariance:	$\mathrm{Im}\,(C_j C_k^* + C_j' C_k'^*) = 0$,		
and	$\mathrm{Re}\,(C_j C_k'^* + C_j' C_k^*) = 0$,		all j and k.

For a single interaction term, (*b*) is always satisfied, and (*a*) and (*c*) are equivalent. The general results agree with the interpretation given by LEE, OEHME and YANG [5] of the various terms in equation (A.6) of reference [3].

It is interesting to investigate the significance of the first of conditions (*b*). Since its violation is not inconsistent with condition (*a*), one would expect terms proportional to $\mathrm{Im}\,(C_j C_k^* + C_j' C_k'^*)$ to enter the predictions of processes which would not detect parity non-conservation, such as for example β-ν angular correlation. LEE and YANG [3] do not find such terms, whereas MORITA [8] does. I have checked this calculation, and am in agreement with

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[8] M. MORITA: *Prog. Theor. Phys.*, **10**, 364 (1953).

MORITA. Equation (A.4) of reference (3) should be replaced by

$$(5) \qquad a\xi = \frac{1}{3}\Big[|C_{\mathrm{T}}|^2 - |C_{\mathrm{A}}|^2 + |C'_{\mathrm{T}}|^2 - |C'_{\mathrm{A}}|^2 - \frac{iZe^2}{\hbar cp}(C_{\mathrm{T}}C^*_{\mathrm{A}} - C^*_{\mathrm{T}}C_{\mathrm{A}} + C'_{\mathrm{T}}C'^*_{\mathrm{A}} - C'^*_{\mathrm{T}}C'_{\mathrm{A}})\Big]|M_{\mathrm{GT}}|^2 - \Big[|C_{\mathrm{S}}|^2 - |C_{\mathrm{V}}|^2 + |C'_{\mathrm{S}}|^2 - |C'_{\mathrm{V}}|^2 - \frac{iZe^2}{\hbar cp}(C^*_{\mathrm{S}}C_{\mathrm{V}} - C_{\mathrm{S}}C^*_{\mathrm{V}} + C'^*_{\mathrm{S}}C'_{\mathrm{V}} - C_{\mathrm{S}}C'^*_{\mathrm{V}})\Big]|M_{\mathrm{F}}|^2 .$$

As with equation (A.6) of reference (3), the « irreversible » contribution to (5) vanishes in the absence of the Coulomb distortion of the electron wave-function. This in an example of a general theorem stated in the Appendix.

3. – Nuclear β decay. Further considerations.

Since the change of coupling constants defined by (4) can be brought about by a unitary transformation, the physical predictions of the theory can depend on these coupling constants only through combinations invariant under (4). To lowest order in perturbation theory, these must be bilinear. The only such bilinear combinations are

$$(6) \qquad C_jC^*_k + C'_jC'^*_k \quad \text{and} \quad C_jC'^*_k + C'_jC^*_k .$$

Comparison with conditions (*a*)–(*c*) of the previous section shows which processes could be used to detect the various types of combination. For example, Im $(C_jC^*_k + C'_jC'^*_k)$ might be found by observing a process which could detect simultaneous break-down of time-reversal invariance and particle-antiparticle conjugation invariance, while not detecting parity non-conservation. The electron-neutrino angular correlation is such a process.

Use of (6) greatly reduces the labour of computation with the theory. For example, it is readily seen, either directly or as in the Appendix, that only Re $(C_jC^*_k + C'_jC'^*_k)$ contributes to spectrum shape correction factors: these may therefore be obtained from the standard references by making the substitution

$$C_jC_k \to \mathrm{Re}\,(C_jC^*_k + C'_jC'^*_k) .$$

Similarly equation (5) above can be written down from Morita's results. For first forbidden transitions, MORITA (9) has given the β-ν angular correlation for the case of real coupling constants; to extend these results it is necessary

(9) M. MORITA: *Prog. Theor. Phys.*, **9**, 345 (1953).

to find the dependence on Im $(C_j C_k^*)$, which can then be immediately extended to give that on Im $(C_j C_k^* + C_j' C_k'^*)$. Similar remarks hold for β-γ angular correlation. For allowed transitions of light nuclei at least, terms proportional to Im $(C_j C_k^* + C_j' C_k'^*)$ will be unimportant because of the theorem in the Appendix.

A further symmetry allows some at least of the terms arising in predictions for n-th forbidden transitions to be derived from results for the $(n-1)$-th forbidden transitions. The results of calculations normally leave nuclear matrix elements as parameters which multiply factors derived solely from the lepton wave functions. The covariants constructed from lepton variables which are associated with a product of nuclear matrix elements $(\int \mathcal{O})(\int \mathcal{O}')^*$ are identical with those associated with the products $(\int \mathcal{O}\gamma_5)(\int \mathcal{O}'\gamma_5)^*$. Hence the multiplying factors finally obtained must also be the same for the two products. For example, the β-ν angular correlation associated with $(|C_V|^2 + |C_V'|^2)\,|\int \boldsymbol{\alpha}|^2$ is identical with that associated with $(|C_A|^2 + |C_A'|^2)\,|\int \boldsymbol{\sigma}|^2$; the Fierz interference term associated with $\mathrm{Re}\,(C_T C_A^* + C_T' C_A'^*)[(\int \beta\boldsymbol{\sigma}) \cdot (\int \boldsymbol{\sigma})^* + \text{c.c.}]$ is identical with that associated with $-\mathrm{Re}\,(C_T C_V^* + C_T' C_V'^*)[(\int \beta\boldsymbol{\alpha}) \cdot (\int \boldsymbol{\alpha})^* + \text{c.c.}]$. The sign-difference arises because C_A is defined to be the coefficient of $+\int \psi_p^\dagger \boldsymbol{\sigma} \psi_n \psi_e^\dagger \boldsymbol{\sigma} \psi_\nu$ whereas C_V is the coefficient of $-\int \psi_p^\dagger \boldsymbol{\alpha} \psi_n \psi_e^\dagger \boldsymbol{\alpha} \psi_\nu$. This symmetry can be used either to reduce the labour of calculations or as a consistency check.

This symmetry can also be used to resolve the difference between the cross-terms involving the (P, T) and (A, T) interaction mixtures as given by SMITH [10] and by PURSEY [11].

Use of the symmetry shows that both are correct but both incomplete. PURSEY omitted terms which would contribute only to $0 \to 0$ (yes) transitions, whereas these are the only terms given by SMITH. ALAGA, KOFOED-HANSEN and WINTHER [12] have shown that terms such as those given by PURSEY may not be entirely negligible, although details of the correction factors will be changed when the velocity-dependent matrix elements are reduced to non-relativistic form.

4. – μ meson decay. Invariance conditions.

For the case of μ-meson decay, the results depend on whether two neutrinos (or antineutrinos) or a neutrino and an antineutrino are emitted. These two cases will be called 2ν emission and $\nu\bar{\nu}$ emission respectively. For both, the

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[10] A. M. SMITH: *Phys. Rev.*, **82**, 955 (1951).

[11] D. L. PURSEY: *Phil. Mag.*, **42**, 1193 (1951).

[12] G. ALAGA, O. KOFOED-HANSEN and A. WINTHER: *K. Danske Vidensk. Selsk., Mat.-fys. Medd.*, **28**, No. 3 (1953).

Fermi interaction terms responsible for the decay will be assumed written as scalar products of a covariant constructed from muon and electron fields with one constructed solely from the neutrino field. The changes in the coupling constants (analogous to (4)) generated by the unitary transformation (3) may be expressed as follows:

2ν *emission*:

$$(7)\qquad \begin{cases} C_j + C_j' = (B_j + B_j') \exp[2i\theta]\,, & \\ C_j - C_j' = (B_j - B_j') \exp[2i\varphi]\,, & j = \mathrm{S,\ T,\ P}, \\ C_j = B_j \exp[i(\theta+\varphi)]\,, \quad C_j' = B_j' \exp[i(\theta+\varphi)]\,, & j = \mathrm{V,\ A}\,. \end{cases}$$

$\nu\bar{\nu}$ *emission*:

$$(8)\qquad \begin{cases} C_j + C_j' = (B_j + B_j') \exp[i(\theta-\varphi)], & \\ C_j - C_j' = (B_j - B_j') \exp[-i(\theta-\varphi)], & j = \mathrm{S,\ T,\ P}, \\ C_j = B_j\,, \qquad C_j' = B_j'\,, & j = \mathrm{V,\ A}\,. \end{cases}$$

For the special case $\varphi = \pi$, $\theta = 0$, (3) becomes $\psi_\nu \to \gamma_5 \psi_\nu$, and (7) and (8) show that for 2ν emission, S. T and P are invariant while V and A change sign, whereas for $\nu\bar{\nu}$ emission, V and A are invariant and S, T and P change sign. This agrees with the results of SALAM [13].

While (7) and (8) give the coupling constant changes generated by the most general unitary transformation (3), it is possible to generate further changes by a phase transformation of the electron or muon fields. This would introduce an additional phase factor, $e^{i\alpha}$ say, in the right-hand side of all the equations (7) and (8). It is seen that the general structure of equations (7) is unaltered by this, while the structure of equations (8) becomes similar to that of equations (7). It follows that the general invariance conditions and the invariant coupling constant combinations will be the same for both 2ν emission and $\nu\bar{\nu}$ emission. The invariance conditions can be expressed as:

(a') *P* invariance:

$$C_j C_k'^* = 0\,, \qquad j,\ k = \mathrm{V,\ A},$$

$$C_j C_k'^* + C_j' C_k^* = 0\,, \qquad j,\ k = \mathrm{S,\ T,\ P};$$

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[13] A. SALAM: *Nuovo Cimento*, **5**, 299 (1957).

(b') T invariance:

$$\mathrm{Im}\,(C_j C_k^*) = \mathrm{Im}\,(C_j' C_k'^*) = \mathrm{Im}\,(C_j' C_k^*) = 0\,, \qquad j, k = \mathrm{V}, \mathrm{A},$$

$$\mathrm{Im}\,(C_j C_k^* + C_j' C_k'^*) = 0\,,$$
$$\mathrm{Im}\,(C_j C_k'^* + C_j' C_k^*) = 0\,, \qquad j, k = \mathrm{S}, \mathrm{T}, \mathrm{P},$$

$$\mathrm{Im}\left[\begin{Bmatrix}(C_j C_k - C_j' C_k')\\(C_j C_k' - C_j' C_k)\end{Bmatrix}\begin{bmatrix}C_l^* C_m^*\\C_l^* C_m'^*\\C_l'^* C_m'^*\end{bmatrix}\right] = 0\,, \qquad \begin{matrix} j, k = \mathrm{S}, \mathrm{T}, \mathrm{P},\\ l, m = \mathrm{V}, \mathrm{A}\end{matrix}$$

(c') C invariance:

$$\mathrm{Im}\,(C_j C_k^*) = \mathrm{Im}\,(C_j' C_k'^*) = \mathrm{Re}\,(C_j' C_k^*) = 0\,, \qquad j, k = \mathrm{V}, \mathrm{A},$$

$$\mathrm{Im}\,(C_j C_k^* + C_j' C_k'^*) = 0\,,$$
$$\mathrm{Re}\,(C_j C_k'^* + C_j' C_k^*) = 0\,, \qquad j, k = \mathrm{S}, \mathrm{T}, \mathrm{P},$$

$$\mathrm{Im}\left[(C_j C_k - C_j' C_k')\begin{Bmatrix}C_l^* C_m^*\\C_l'^* C_m'^*\end{Bmatrix}\right] = 0\,,$$
$$\mathrm{Im}\,[(C_j C_k' - C_j' C_k) C_l^* C_m^*] = 0\,, \qquad j, k = \mathrm{S}, \mathrm{T}, \mathrm{P},$$
$$\mathrm{Re}\left[(C_j C_k' - C_j' C_k)\begin{Bmatrix}C_l^* C_m^*\\C_l'^* C_m'^*\end{Bmatrix}\right] = 0\,, \qquad l, m = \mathrm{V}, \mathrm{A}\,.$$
$$\mathrm{Re}\,[(C_j C_k - C_j' C_k') C_l^* C_m'^*] = 0\,.$$

The first of the conditions (a') recognises that the conditions on the coupling constants for V and A for P invariance are not just $C_{\mathrm{V}}' = C_{\mathrm{A}}' = 0$, but are either $C_{\mathrm{V}}' = C_{\mathrm{A}}' = 0$ or $C_{\mathrm{V}} = C_{\mathrm{A}} = 0$, because of the arbitrariness in the phases associated with the electron and muon fields for the transformation. For the lasts sets of conditions (b') and (c'), one must take in turn each of the first factors with each of the second factors, making six types of equation in all.

5. – μ meson decay. Further considerations.

As for nuclear β-decay, physically observable processes can depend on the coupling constants only through combinations which are invariant under transformations of the type of (7). To lowest order in perturbation theory,

these combinations are bilinear. The only bilinear invariants are

$$(9) \qquad \begin{cases} C_j C_k^* , \quad C_j C_k'^* , \quad C_j' C_k'^* , & j, k = \mathrm{V}, \mathrm{A}, \\ C_j C_k^* + C_j' C_k'^* , \quad C_j C_k'^* + C_j' C_k^* , & j, k = \mathrm{S}, \mathrm{T}, \mathrm{P}. \end{cases}$$

Hence for μ-meson decay with the ordering of fields indicated, to lowest order in H_{int} there can be no interference effects between the groups of interaction terms (V, A) and (S, T, P).

The quadrilinear invariants involved in the last sets of conditions in (b') and (c') cannot enter the theory in this order, and can be detected only in processes involving at least two Fermi interactions. Such processes are expected to be so weak as to be undetectable.

6. – Other interactions.

Considerations for the interaction responsible for μ-capture in nuclei will be the same as in Sect. **2** and **3**, and any other interactions involving neutrinos can be treated similarly. Strictly, one should also consider the conditions that a single choice of θ and φ in (3) should simultaneously reduce all interactions to forms which are trivially P invariant, T invariant, C invariant. However, if for example the nuclear β-decay interaction and the μ-meson decay interaction could each by itself, but not both simultaneously, be put in trivially T invariant form, detection of the consequences of non-invariance would require processes depending on interferences between nuclear β-decay and μ-meson decay, and these would be too weak to make their observation practicable.

7. – Neutrino gauge transformations.

SALAM [13] has shown that if the bare neutrino mass is zero and if the complete Lagrangian is invariant under the substitution $\psi_\nu \to \gamma_5 \psi_\nu$, then the neutrino self-mass must also be zero. By postulating invariance of the Lagrangian under this substitution, he showed that the interaction in μ-meson decay with 2ν emission must be a combination of S and P, while that for $\nu\bar{\nu}$ emission must be a combination of V and A. However, the discussion in Sect. **4** above shows that the distinction between 2ν emission and $\nu\bar{\nu}$ emission is less clear than Salam's argument would lead one to believe. Indeed, there are a number of other invariance postulates, admittedly less natural than Salam's, which are equally successful as regards the vanishing of the neutrino self-mass but exactly reverse Salam's conclusions with regard to the μ-decay interaction; that is, they give a combination of V and A for 2ν emission and

of S, T and P for $\nu\bar{\nu}$ emission. One example of such a postulate is the requirement of invariance under the simultaneous substitutions

$$\psi_e \to -\psi_e\,, \qquad \psi_\nu \to -\gamma_5\psi_\nu\,. \tag{10}$$

In Salam's case, the identity of the emitted neutrinos in 2ν emission caused the contribution from T to vanish: here, it is V and A′ which vanish. It is not possible in this theory to decide on the basis of the Michel parameter alone whether the decay involves 2ν emission or $\nu\bar{\nu}$ emission.

Among the other consequences of the theory it is found that the relative signs of odd and even terms in the interaction are different for nuclear β-decay and for μ-capture, (provided that it is a neutrino that is emitted, as in electron capture, and not an antineutrino). The theory is not consistent with a single two-component neutrino; rather there must be two two-component neutrinos (not related by particle-antiparticle conjugation), one of which participates in nuclear β-decay, the other in μ-meson capture, and both in μ-meson decay.

8. – Discussion.

It is interesting to examine more closely the arguments used in this paper. The Lagrangian of the theory may be written as

$$L = L_0 + L_i(C)\,, \tag{11}$$

where L_0 includes all parts of L independent of the neutrino field and the Fermi coupling constants, together with the free neutrino Lagrangian for the *observable* neutrino mass (assumed zero), and $L_i(C)$, a function of the Fermi coupling constants C, includes all interaction terms in L involving the neutrino field together with any necessary counter-terms such as those required to ensure that the observable neutrino mass vanishes. This of course assumes that the complete theory is renormalizable at least in principle, if not in practice in the perturbation expansion.

The functional dependence of the counter-terms on C will be completely fixed by the requirement that they cancel out any corrections, depending on C, to observable quantities occurring in L_0. Thus if the theory is changed by changing the Fermi coupling constants only but not changing any observable quantity in L_0, the counter terms must be modified only to the extent of writing them as the same functions but of the new coupling constants. That is, the new theory obtained by the substitution $C \to B$, while all obser-

vables in L_0 keep the same values, will have for Lagrangian

$$L_0 + L_i(B) . \tag{12}$$

Any observable prediction of the theory with Lagrangian (11) can be expressed in terms of functions of the C; $a(C)$ will be used to denote a typical such observable. For the Lagrangian (12), the corresponding observable is $a(B)$.

Now (3) is a unitary transformation whose effect is just a special case of the transformation from (11) to (12). As a unitary transformation, it must leave unaltered the result of any measurement, so that

$$a(C) = a(B) .$$

Hence the results of experiments can depend on the coupling constants only through combinations which are invariant under the changes generated by (3).

It is essential for the above argument that L_0 should be unaltered by (3), and therefore that the observable neutrino mass should be zero, for otherwise the functional form as well as the arguments of $a(C)$ would be altered in the transition. It is not necessary that the bare neutrino mass or the neutrino self-mass be zero, since they are not observable: indeed the bare mass is related simply to the counter terms required to cancel the self-mass, and therefore is a function of the coupling constants and appears in L_i. That these counter terms are changed by (3) only through the change in the values of the coupling constants is ensured by the facts that the zero neutrino mass in L_0 is an observable, and that (3) is a unitary transformation.

* * *

It is a pleasure to thank Professor W. Pauli, Professor N. Kemmer, Dr. J. C. Polkinghorne and Mr. S. Kahana for valuable discussions.

Appendix

Theorem. – If the total hamiltonian can be written

$$H = H_s + H_w ,$$

where H_s is invariant under P, T and C separately, and H_w is invariant under P, and if a stationary state A of H_s decays through H_w to a state B which con-

tains only free particles, then to lowest order in H_w there are no interference effects in the decay of A between the parts of H_w which commute and anti-commute with C.

Proof. – The proof is illustrated for the case when A has zero angular momentum. The general proof is straight-forward.

The *observable* final state B is a linear combination of the outgoing parts of a number of stationary states $B_1, B_2, \ldots$ of H_s, with coefficients proportional to $\langle B_i|H_w|A\rangle$, $i = 1, 2, \ldots$. If the states B_i contain only free particles, and if the phases are chosen so that

$$|TB_i\rangle = |B_i\rangle\,, \qquad |TA\rangle = |A\rangle \tag{A.1}$$

then the outgoing parts of different B_i have real relative amplitudes.

It is easily seen by the methods of reference [4] that

$$\begin{cases} \langle B_i|H_1|A\rangle & \text{is real}\,, \\ \langle B_i|H_2|A\rangle & \text{is pure imaginary}\,, \end{cases} \tag{A.2}$$

where

$$H_1 = \tfrac{1}{2}(H_w + C^{-1}H_wC) \quad \text{and} \quad H_2 = \tfrac{1}{2}(H_w - C^{-1}H_wC) \tag{A.3}$$

are the parts of H_w which commute and anticommute with C respectively.

The theorem now follows.

The result found in Sect. 2 does not depend directly on this theorem, but depends instead on the

Corollary 1. – The result of the theorem remains true if each state B_i can be written as a product

$$|B_i\rangle = |D\rangle|E_i\rangle\,,$$

where $|D\rangle$ is the same for all states $|B_i\rangle$ and $|E_i\rangle$ contains only free particles.

The proof of this is trivial.

The result on the shape correction factors quoted in Sect. 3 follows from

Corollary 2. – If $H = H_s + H_w$ satisfies the same conditions as in the theorem, the only processes in the decay of A into B which can detect interferences between H_1 and H_2 defined in (A.3), at least to lowest order in H_w, are processes involving interference between different partial waves of the decay products, i.e. between different B_i.

RIASSUNTO (*)

Si esaminano le conseguenze della massa osservabile annullantesi del neutrino sulle condizioni dell'invarianza delle interazioni di Fermi rispetto a P, T e C. Si dà una piccola correzione della formula data da LEE e YANG per la correlazione angolare β–ν. Si ottengono risultati che riducono il lavoro necessario per estendere i calcoli attuali alle interazioni non invarianti rispetto a P, T e C. Si chiariscono alcuni punti oscuri nella letteratura esistente sui fattori di correzione alla forma degli spettri β. Si mostra che nel decadimento dei mesoni μ fino all'ordine minimo non ci può essere interferenza tra il gruppo delle interazioni (V, A) e il gruppo (S, T, P). Si propone un'alternativa all'ipotesi di Salam sulla gauge del neutrino. Si discute la fondatezza del metodo usato nel lavoro. In appendice si espone un teorema generale sulle interazioni deboli invarianti rispetto a P, ma non rispetto a T o C assieme a due corollari e se ne delinea la prova.

(*) *Traduzione a cura della Redazione.*

La struttura del cloruro di (di)acetamide-cadmio (*).

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Riassunto. — I risultati dello studio strutturale del cloruro di (di)acetamide-cadmio, $CdCl_2(CH_3CONH_2)_2$, mostrano che l'atomo di cadmio coordina planarmente due atomi di cloro e due molecole di acetamide; gli atomi coordinati giacciono nei vertici di un quadrato leggermente distorto. La coordinazione diventa ottaedrica se si considera che ogni atomo di cloro è quasi alla stessa distanza da due atomi di cadmio contigui.

1. – Introduzione.

In una nota in corso di stampa sulla *Ricerca Scientifica* abbiamo riferito i risultati dell'indagine roentgenografica preliminare condotta sul cloruro di (di)acetamide-cadmio, $CdCl_2(CH_3CONH_2)_2$, che cristallizza nel sistema monoclino (gruppo spaziale $C_{2h}(5) - P2_1/c$) con le costanti: $a = 3.79$ Å, $b = 9.62$ Å, $c = 12.69$ Å, $\beta = 97.7°$; la cella elementare contiene due molecole.

Il valore particolarmente piccolo della costante a permette di prevedere che la coordinazione attorno al cadmio deve essere planare, come si deduce anche dalla simmetria del gruppo spaziale. Infatti in questo tutte le posizioni di due punti, in una delle quali si deve trovare il cadmio che è baricentro della molecola, giacciono su centri di simmetria. Poichè esse sono equivalenti fra di loro abbiamo posto i due atomi di cadmio in $a)$ 0, 0, 0; 0, $\frac{1}{2}$, $\frac{1}{2}$ perchè ciò fa sì che la maggior parte dei fattori di struttura risulti positiva a causa del contributo preponderante degli atomi di cadmio.

I risultati ottenuti dallo studio delle proiezioni della densità elettronica sul piano normale all'asse [100] e su quello (010) hanno confermato le previsioni fatte.

(*) Lavoro eseguito con il contributo del C.N.R.

2. – Determinazione degli F_o.

La misura delle intensità delle riflessioni $(0kl)$ ed $(h0l)$ è stata compiuta sui relativi fotogrammi di Weissenberg ottenuti con la camera integratrice di Wiebenga e Smits [1] (radiazione usata $K\alpha$ del rame), applicando la tecnica del film multiplo. Per facilitare la correzione degli effetti dovuti all'assorbimento ($\mu = 242$ cm^{-1}) abbiamo reso i preparati all'incirca cilindrici in entrambi i casi (diametro medio ~ 0.02 cm).

La stima delle intensità è stata compiuta visualmente per confronto con scale di riferimento opportunamente ottenute dagli stessi preparati mediante serie di fotogrammi integrati di Weissenberg a piccola oscillazione con esposizioni varianti linearmente. Lo scostamento medio di due serie di stime dal valore medio è risultato del 4.9% per le 88 riflessioni indipendenti di tipo $(0kl)$ osservate (possibili 157). Nel caso delle riflessioni $(h0l)$ se ne sono osservate 41 delle 60 indipendenti possibili; due serie di stime hanno dato valori che in media si scostano dal valore medio del 3%.

Dalle intensità si è risaliti ai fattori di struttura osservati, tenendo conto dei fattori di Lorentz, di polarizzazione e di assorbimento ed applicando il metodo di Wilson [2], limitatamente ai fattori di struttura delle riflessioni $(0kl)$, per portarli all'incirca in scala assoluta. L'ulteriore affinamento è stato condotto successivamente per confronto con i valori calcolati; il fattore di correzione risultato dall'ultimo confronto riferito ai valori ottenuti dall'applicazione del metodo di Wilson è $K = 0.966$, mentre il coefficiente di temperatura medio $B = 2.5$ Å^2 non ha subito variazioni.

I fattori di struttura delle riflessioni $(h0l)$ sono stati portati in scala assoluta dopo completamento dello studio della proiezione sul piano normale a [100], per confronto fra i fattori di struttura delle riflessioni $(00l)$ che sono comuni alle due zone. Il coefficiente di temperatura medio per queste riflessioni è risultato $B = 2.2$ Å^2.

3. – Proiezione di Patterson sul piano normale a [100].

Nella Fig. 1 è riportata la proiezione $P(V, W)$ limitatamente all'unità asimmetrica che si estende per $\frac{1}{2}b$ ed $\frac{1}{2}c \sin\beta$. In essa, oltre al picco O all'origine, si osserva un picco assai elevato A, in $\frac{1}{2}, \frac{1}{2}$, evidentemente dovuto all'interazione Cd(0, 0, 0) – Cd($0, \frac{1}{2}, \frac{1}{2}$); i picchi B e C di peso all'incirca uguale devono riferirsi alle interazioni Cd – Cl, mentre gli altri contrassegnati con le lettere

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[1] E. H. Wiebenga D. W. Smits: *Acta Cryst.*, **3**, 265 (1950).

[2] A. J. C. Wilson: *Nature*, Londra, **150**, 152 (1942).

D, *E*, *F* e *G*, che presentano pesi non molto diversi fra di loro ma assai inferiori a quelli dei precedenti, possono essere attribuiti alle interazioni fra atomo di cadmio ed atomi leggeri delle molecole di acetamide.

Fig. 1. – Proiezione di Patterson sul piano normale a [100]. Equidistanze arbitrarie.

Limitando la nostra attenzione, nei primi stadi della ricerca, alla distribuzione degli atomi più pesanti, abbiamo assunto le seguenti coordinate di partenza per tali atomi:

$$2\ \mathrm{Cd} \quad \text{in } a) \quad 0,\ 0,\ 0;\ 0,\ \tfrac{1}{2},\ \tfrac{1}{2}$$

$$4\ \mathrm{Cl} \quad \text{in } e) \quad x,\ y,\ z;\ \bar{x},\ \bar{y},\ \bar{z};$$

$$\bar{x},\ \tfrac{1}{2}+y,\ \tfrac{1}{2}-z;\ \ x,\ \tfrac{1}{2}-y,\ \tfrac{1}{2}+z$$

con $z=0$ e $y=0.189$

corrispondenti ad una distribuzione che richiede l'estinzione delle riflessioni di tipo $(0kl)$ quando $k+l=2n+1$. Questa condizione è approssimativamente riscontrata poichè, mentre delle 86 riflessioni con $k+l=2n$ indipendenti possibili ne sono state osservate 82, delle 71 riflessioni con $k+l=2n+1$ indipendenti possibili ne sono state osservate soltanto 6 di piccola intensità; la comparsa di queste ultime può quindi essere dovuta al fatto che la coordinata z degli atomi di cloro non sia esattamente nulla e che gli atomi leggeri si trovino in posizione generale con z ed y diversi da zero.

Per un primo calcolo dei fattori di struttura abbiamo inoltre supposto che i picchi *D* ed *E* nella proiezione di Patterson fossero dovuti alle interazioni fra cadmio e due diversi atomi della molecola di acetamide evidentemente posti nella posizione generale *e*) e che indichiamo rispettivamente con i simboli 1 e 3; le coordinate per essi dedotte sono risultate:

$$(1) \quad y=0 \quad \text{e} \quad z=0.167\ ,$$

$$(3) \quad y=0 \quad \text{e} \quad z=0.349\ .$$

Il calcolo dei fattori di struttura limitato ai contributi degli atomi considerati ed alle sole riflessioni con $k+l=2n$ che presentano $\sin^2\vartheta \leqslant 0.4$, ha dato un valore dell'indice $R(0kl)=\sum\big||F_o|-|F_c|\big| \big/ \sum|F_o|=0.183$; valore che indica come almeno nelle linee generali l'interpretazione data della proiezione di Patterson fosse corretta.

4. – Proiezione di Fourier sul piano normale a [100].

Una prima proiezione di Fourier è stata ottenuta utilizzando soltanto i fattori di struttura delle riflessioni con $k+l = 2n$ che sono risultati tutti positivi; questa ci ha permesso di intravedere la posizione di tutti gli atomi leggeri.

L'ulteriore affinamento delle coordinate è stato ottenuto con proiezioni di Fourier, con sintesi $(F_0 - F_c)$ ed infine con l'applicazione del metodo dei minimi quadrati. L'indice di attendibilità, calcolato per le riflessioni osservate, è risultato al termine delle varie operazioni di affinamento $R(0kl) = 0.079$.

Poichè non ci è stato possibile distinguere fra di loro i tre atomi C, O, N periferici dell'acetamide abbiamo continuato a contrassegnarli con i simboli 1, 2, 3; le coordinate che hanno dato il miglior accordo fra i fattori di struttura osservati e quelli calcolati (v. Tabella I) sono:

	y	z
Cd	0	0
Cl	0.187	0.011
C	0.065	0.259
1	0.973	0.177
2	0.187	0.245
3	0.028	0.367

Nella Fig. 2 *b*) è rappresentata la proiezione di Fourier definitiva.

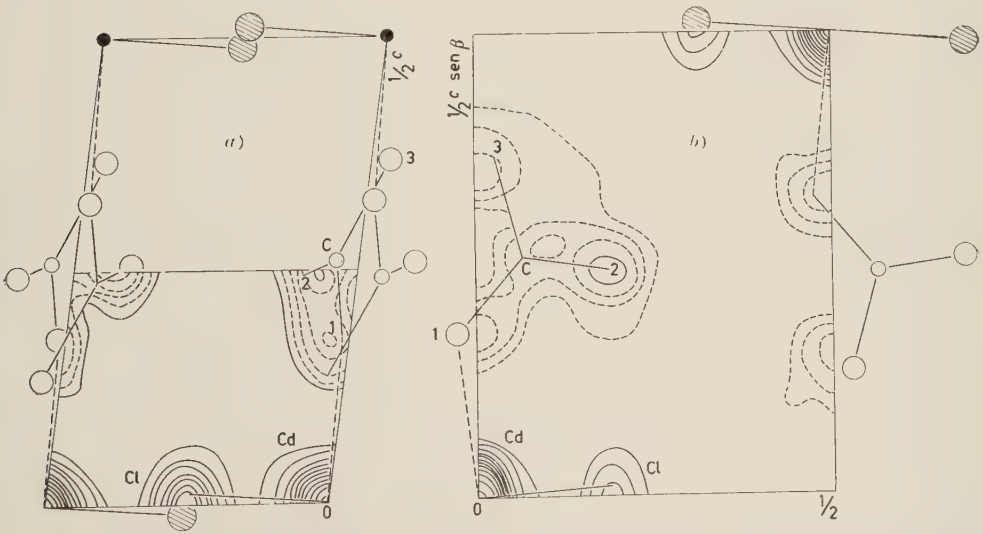

Fig. 2. – *a*) proiezione di Fourier su (010); prima linea a 5 *e* Å^{-2}, equidistanze: fra le linee tratteggiate 1 *e* Å^{-2}, fra le linee intera 5 *e* Å^{-2}; *b*) proiezione di Fourier sul piano normale a [100]; prima linea tratteggiata a 2 *e* Å^{-2}, prima linea intera a 5 *e* Å^{-2}; equidistanze come in *a*).

TABELLA I. – *Fattori di struttura osservati e calcolati per le riflessioni* (0 *k l*).

0 *k l*	$\|F_0\|$	F_c	0 *k l*	$\|F_0\|$	F_c	0 *k l*	$\|F_0\|$	F_c
0 0 2	79	82	0, 3, 15	26	26	0 7 8	—	— 5
0 0 4	113	144	0 4 0	93	100	0 7 9	38	39
0 0 6	102	102	0 4 1	14	12	0, 7, 10	—	— 22
0 0 8	106	109	0 4 2	65	64	0, 7, 11	42	40
0, 0, 10	71	61	0 4 3	—	3	0, 7, 12	—	— 13
0, 0, 12	87	84	0 4 4	66	55	0, 7, 13	25	39
0, 0, 14	55	46	0 4 5	—	16	0 8 0	—	9
0, 0, 16	69	70	0 4 6	78	76	0 8 1	—	4
0 1 1	82	112	0 4 7	—	24	0 8 2	34	38
0 1 2	—	— 2	0 4 8	56	53	0 8 3	—	— 7
0 1 3	89	96	0 4 9	—	13	0 8 4	—	6
0 1 4	—	— 13	0, 4, 10	56	50	0 8 5	—	0
0 1 5	102	110	0, 4, 11	—	14	0 8 6	36	39
0 1 6	—	— 20	0, 4, 12	54	47	0 8 7	—	11
0 1 7	67	73	0, 4, 13	—	33	0 8 8	—	10
0 1 8	—	— 11	0, 4, 14	46	42	0 8 9	—	— 11
0 1 9	63	52	0, 4, 15	—	15	0, 8, 10	31	36
0, 1, 10	—	— 24	0 5 1	103	106	0, 8, 11	—	— 4
0, 1, 11	78	80	0 5 2	—	20	0, 8, 12	—	14
0, 1, 12	—	— 15	0 5 3	96	94	0 9 1	40	39
0, 1, 13	53	49	0 5 4	—	— 7	0 9 2	—	19
0, 1, 14	—	— 25	0 5 5	104	105	0 9 3	35	38
0, 1, 15	49	42	0 5 6	—	0	0 9 4	—	— 5
0 2 0	78	89	0 5 7	76	76	0 9 5	39	39
0 2 1	37	— 32	0 5 8	—	16	0 9 6	—	10
0 2 2	48	33	0 5 9	76	72	0 9 7	31	35
0 2 3	22	15	0, 5, 10	—	2	0 9 8	—	17
0 2 4	26	23	0, 5, 11	80	73	0 9 9	33	38
0 2 5	32	— 32	0, 5, 12	—	13	0, 9, 10	—	17
0 2 6	65	64	0, 5, 13	65	63	0, 9, 11	27	32
0 2 7	—	10	0, 5, 14	—	2	0, 10, 0	70	66
0 2 8	37	34	0 6 0	94	105	0, 10, 1	—	15
0 2 9	22	— 31	0 6 1	—	— 11	0, 10, 2	67	68
0, 2, 10	40	36	0 6 2	81	82	0, 10, 3	—	— 12
0, 2, 11	—	— 7	0 6 3	—	2	0, 10, 4	62	68
0, 2, 12	40	34	0 6 4	83	79	0, 10, 5	—	17
0, 2, 13	—	— 19	0 6 5	—	— 21	0, 10, 6	60	60
0, 2, 14	35	32	0 6 6	85	83	0, 10, 7	—	9
0, 2, 15	—	— 11	0 6 7	—	13	0, 10, 8	54	60
0 3 1	36	33	0 6 8	71	71	0, 10, 9	—	10
0 3 2	22	22	0 6 9	—	— 31	0, 11, 1	69	72
0 3 3	36	29	0, 6, 10	65	62	0, 11, 2	—	11
0 3 4	—	— 4	0, 6, 11	—	— 9	0, 11, 3	69	70
0 3 5	51	48	0, 6, 12	61	60	0, 11, 4	—	— 12
0 3 6	—	0	0, 6, 13	—	— 10	0, 11, 5	66	65
0 3 7	29	29	0, 6, 14	43	49	0, 11, 6	—	— 4
0 3 8	—	16	0 7 1	48	46	0, 11, 7	57	64
0 3 9	18	17	0 7 2	—	13	0, 12, 0	41	39
0, 3, 10	—	1	0 7 3	47	42	0, 12, 1	—	4
0, 3, 11	51	48	0 7 4	—	— 19	0, 12, 2	45	45
0, 3, 12	—	13	0 7 5	51	50	0, 12, 3	—	— 9
0, 3, 13	37	26	0 7 6	—	— 17			
0, 3, 14	—	2	0 7 7	39	38			

5. – Proiezione di Fourier su [010].

La prima proiezione della densità elettronica $\varrho(X, Z)$ è stata ottenuta assumendo tutti i segni dei fattori di struttura positivi; questa si è dimostrata anche definitiva perchè non si è avuto in seguito alcuna variazione di segno. La proiezione ottenuta è rappresentata nella Fig. 2 *a*); in essa risultano risolti soltanto gli atomi di cadmio, infatti due atomi di cloro di molecole contigue sono quasi sovrapposti, come pure risultano sovrapposti gli atomi delle molecole di acetamide, fatto già prevedibile dall'esame della proiezione di $\varrho(X, Z)$.

L'affinamento delle coordinate è stato ottenuto come nel caso della proiezione precedente, applicando cioè il metodo delle sintesi $(F_0 - F_c)$ e dei minimi quadrati, tenendo naturalmente conto delle sovrapposizioni.

Il valore dell'indice $R(h0l)$ ottenuto al termine di queste operazioni è risultato 0.146 e le relative coordinate atomiche sono:

	x	z
Cd	0	0
Cl	0.491	0.009
C	0.083	0.266
1	0.025	0.173
2	0.197	0.237
3	0.936	0.362

TABELLA II. – *Fattori di struttura osservati e calcolati per le riflessioni* ($h\,0\,l$).

$h\ 0\ l$	$\lvert F_0\rvert$	F_c	$h\ 0\ l$	$\lvert F_0\rvert$	F_c	$h\ 0\ l$	$\lvert F_0\rvert$	F_c
0 0 2	94	82	2 0 8	75	67	$\bar{1}$, 0, 16	27	45
0 0 4	128	143	2, 0, 10	74	65	$\bar{2}$ 0 2	97	112
0 0 6	106	101	2, 0, 12	59	62	$\bar{2}$ 0 4	95	78
0 0 8	109	109	3 0 0	—	28	$\bar{2}$ 0 6	120	104
0, 0, 10	70	62	3 0 2	—	13	$\bar{2}$ 0 8	93	79
0, 0, 12	78	84	3 0 4	—	12	$\bar{2}$, 0, 10	79	75
0, 0, 14	39	47	3 0 6	—	43	$\bar{2}$, 0, 12	72	66
0, 0, 16	58	70	3 0 8	—	3	$\bar{2}$, 0, 14	63	50
1 0 0	66	92	3, 0, 10	—	29	$\bar{3}$ 0 2	28	36
1 0 2	—	— 16	4 0 0	106	79	$\bar{3}$ 0 4	—	8
1 0 4	44	36	4 0 2	90	62	$\bar{3}$ 0 6	29	35
1 0 6	34	37	4 0 4	84	72	$\bar{3}$ 0 8	—	25
1 0 8	26	33	4 0 6	85	82	$\bar{3}$, 0, 10	—	32
1, 0, 10	—	16	$\bar{1}$ 0 2	—	2	$\bar{3}$, 0, 12	—	32
1, 0, 12	—	37	$\bar{1}$ 0 4	29	27	$\bar{3}$, 0, 14	—	22
1, 0, 14	—	20	$\bar{1}$ 0 6	56	36	$\bar{4}$ 0 2	101	80
2 0 0	108	125	$\bar{1}$ 0 8	47	41	$\bar{4}$ 0 4	84	64
2 0 2	92	87	$\bar{1}$, 0, 10	—	22	$\bar{4}$ 0 6	95	68
2 0 4	100	86	$\bar{1}$, 0, 12	—	39	$\bar{4}$ 0 8	78	65
2 0 6	101	109	$\bar{1}$, 0, 14	—	21	$\bar{4}$, 0, 10	68	57

Date le numerose sovrapposizioni i risultati ottenuti da questa proiezione sono meno attendibili di quelli dedotti dalla precedente, pertanto abbiamo ritenuto più probabili per le coordinate z i valori da essa ricavati. I fattori di struttura calcolati in base a quest'ultima ammissione, sono confrontati nella Tabella II con quelli osservati; l'indice $R(h0l)$ assume in tal caso il valore 0.154.

6. – Discussione dei risultati.

In base alle coordinate atomiche x dedotte dalla proiezione di $\varrho(X, Z)$ ed a quelle y e z dedotte dalla proiezione di $\varrho(Y, Z)$, si calcolano le seguenti distanze nell'ambito della coordinazione attorno all'atomo di cadmio:

$$\begin{aligned} \mathrm{Cd} - \mathrm{Cl} &= 2{,}58 \text{ Å} \\ \mathrm{Cd} - 1 &= 2.23 \text{ Å} \\ \sphericalangle 1 - \mathrm{Cd} - \mathrm{Cl} &= 96°. \end{aligned}$$

La distanza Cd — Cl ottenuta è intermedia fra quelle che si calcolano per somma dei raggi ionici (2.78 Å) e per somma dei raggi covalenti (2.47 Å).

La distanza fra l'atomo di cadmio posto in 1, 0, 0 dall'atomo di cloro posto in 0.491, 0.187, 0.011 è di 2.66 Å ed è poco diversa da quella fra lo stesso atomo di cloro e l'atomo di cadmio posto all'origine; questo porta a considerare una coordinazione di tipo ottaedrico attorno ad ogni atomo di cadmio nella quale ogni atomo di cloro risulta condiviso da due atomi di cadmio. Nella struttura del composto studiato si possono quindi riscontrare catene di ottaedri distorti, aventi uno spigolo in comune, parallele all'asse [100]. La distanza Cd — 1 trovata è molto vicina alla somma dei raggi covalenti del cadmio e dell'ossigeno (2.22 Å); non riteniamo tuttavia che questo ci permetta di identificare con sicurezza l'atomo 1 con l'ossigeno. Infatti le distanze e gli angoli di legame che si calcolano in seno ad una molecola di acetamide:

$$\begin{aligned} 1 - \mathrm{C} &= 1.35 \text{ Å} \\ 2 - \mathrm{C} &= 1.27 \text{ Å} \\ 3 - \mathrm{C} &= 1.58 \text{ Å} \\ \sphericalangle 1 - \mathrm{C} - 2 &= 121° \\ \sphericalangle 1 - \mathrm{C} - 3 &= 118° \\ \sphericalangle 2 - \mathrm{C} - 3 &= 121° \end{aligned}$$

mentre ci permettono di concludere che l'atomo 3 corrisponde al gruppo metilico, non sono da ritenersi sufficientemente esatte per consentirci di distinguere

l'atomo di ossigeno dal gruppo aminico; la molecola di acetamide risulta planare (*).

Altre distanze interatomiche che si calcolano dalle coordinate trovate sono:

$$\begin{aligned} \mathrm{Cl} - 2 &= 3.29\ \text{Å} \\ 2 - 3' &= 3.58\ \text{Å} \\ 2 - \mathrm{Cl}' &= 3.59\ \text{Å} \\ 2 - 1' &= 4.05\ \text{Å} \\ 3 - \mathrm{Cl}' &= 3.76\ \text{Å} \\ \mathrm{Cl} - 3' &= 4.05\ \text{Å}\ . \end{aligned}$$

Gli atomi contrassegnati con l'apice appartengono alla molecola complessa il cui atomo di cadmio è in 0, $\frac{1}{2}$, $\frac{1}{2}$.

(*) Per comodità di confronto riportiamo i valori ottenuti da F. Senti e D. Harker (*Journ. Am. Chem. Soc.*, **62**, 2008 (1940)) dallo studio dei cristalli romboedrici di acetamide:

$$\mathrm{C{-}CH_3} = 1.51 \pm 0.05\ \text{Å};\quad \mathrm{C{-}NH_2} = 1.38 \pm 0.05\ \text{Å};\quad \mathrm{C{-}O} = 1.28 \pm 0.05\ \text{Å};$$
$$\mathrm{CH_3{-}C{-}NH_2} = 109° \pm 5°;\quad \sphericalangle\, \mathrm{CH_3{-}C{-}O} = 129° \pm 5°;\quad \sphericalangle\, \mathrm{NH_2{-}C{-}O} = 122° \pm 5°.$$

SUMMARY

The results of our structural study on (di)acetamide-cadmium chloride, $CdCl_2 \cdot (CH_3CONH_2)_2$, show that the cadmium atom co-ordinates planarly two chlorine atoms and two acetamide molecules; the co-ordinated atoms lie on the corners of a slightly distorted square. The co-ordination is octahedral if one considers that each chlorine atom is nearly at the same distance from two adjacent cadmium atoms.

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Linear Theory of Synchrotron Oscillations.

II. – Particle Losses During Acceleration and Tolerance Theory (*).

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(ricevuto il 2 Maggio 1957)

Summary. — Various types of synchrotron oscillation perturbations of accelerated particles are examined from the standpoint of the particle losses they produce. A system of tolerances for a prescribed particle loss is set up.

1. – Introduction.

Existing synchrotron oscillation theories ([1,2]) do not go beyond computation of the upbilding of synchrotron oscillations produced by various types of perturbations such as noise modulation of the magnetic field strength, frequency or amplitude of the accelerating voltage as well as ripple modulation of these quantities with synchrotron oscillation frequencies. For tolerance calculations, however, it is not the increment of the synchrotron oscillation amplitude that is required but rather the fraction of particles lost as consequence of action of the perturbations.

It is well known ([1]) that in regions which are remote from the critical point, synchrotron oscillations obey the equation

$$\varphi = \left(\frac{\Omega}{\Omega_1}\right)^{\frac{1}{2}} [c_1 \cos \Omega x + c_2 \sin \Omega x] , \tag{1}$$

(*) Part I of this work is presented in ref. ([1]).

([1]) L. L. Goldin and D. G. Koškarev: *Nuovo Cimento*, **2**, 1251 (1955).

([2]) N. M. Blachman: *Rev. Sci. Instr.*, **21**, 908 (1950).

where $x = pc/E_0$, p is the particle momentum, c the velocity of light, E_0 the rest energy of the accelerated particles, Ω the frequency of the synchrotron oscillations at an arbitrary moment (with respect to variable x), and Ω_1 is the value of Ω at injection.

The relation between x and t (acceleration time) is expressed by the formula

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{ceu \sin \Phi}{E_0 L} \tag{2}$$

In this formula e is the particle charge, eu the maximum energy, which can be acquired by a particle during a revolution, Φ the equilibrium acceleration phase and L the orbith length.

In the absence of perturbations, amplitudes c_1 and c_2 are integrals of motion. We shall call them the particle co-ordinates.

According to linear synchrotron oscillation theory the increments of co-ordinates c_1 and c_2 due to perturbations are the same for any of the particles. Thus, the region occupied by the particles shifts as a whole. As a result of this shift, part of the particle region will be situated in a zone of unstable motion and the corresponding particles will consequently be lost.

Let us consider those particles which at the moment of injection were located in the vicinity of point $c_{1,0}$, $c_{2,0}$. We introduce the function $P_1(c_1, c_2, x)$ which is the probability that the particles will be in te vicinity of point c_1, c_2 at the instant x. In evaluating P_1 it must be taken into account that a particle is lost if it enters at least once the region of unstable motion. This problem, as is well known, can be solved by means of the Fokker-Planck equation. The probability function P for a real particle distribution can be obtained by putting P at the moment of injection equal to the particle distribution at this moment.

The integral $\int P \mathrm{d}c_1 \mathrm{d}c_2$ taken over the region of stable motion yields the mathematical expectation of the amount of remaining particles. If the perturbations do not exactly repeat themselves from cycle to cycle, P may also be considered as the averaged particle distribution in c_1 and c_2, averaged over a set of cycles. Non-linearity of the equations and gas scattering may lead to the result that the perturbations of the various particles cease to be identical; in this case P will describe to a certain degree the particle distribution in c_1 and c_2 even during a single cycle.

2. – Fokker-Planck equation.

Let $P_1(c_1, c_2, x)\mathrm{d}c_1\,\mathrm{d}c_2$ be te probability for a particle to be in the region $c_1 \div (c_1 + \mathrm{d}c_1)$, $c_2 \div (c_2 + \mathrm{d}c_2)$ at a time x. Introduce now the function $W(c_1, c_2, \Delta c_1, \Delta c_2, x)$ such that $W\mathrm{d}x$ is the probability for the co-ordinates

c_1 and c_2 to change by Δc_1 and Δc_2 when x increases by $\mathrm{d}x$. Then

$$\frac{\partial P_1}{\partial x} = \int\int_{-\infty}^{\infty} [P_1(c_1 - \Delta c_1, c_2 - \Delta c_2, x) W(c_1 - \Delta c_1, c_2 - \Delta c_2, \Delta c_1, \Delta c_2, x) - P_1(c_1, c_2, x) W(c_1, c_2, \Delta c_1, \Delta c_2)] \, \mathrm{d}(\Delta c_1) \, \mathrm{d}(\Delta c_2) \,. \tag{3}$$

We expand the integrand into a series and retain terms up to second order of smallness inclusively. We denote the mean square increment of c_1 and c_2 in the interval $\mathrm{d}x$ by $\overline{(\Delta c_1)^2}\,\mathrm{d}x$ and $\overline{(\Delta c_2)^2}\,\mathrm{d}x$. Noting that $\overline{(\Delta c_1)} = \overline{(\Delta c_2)} = \overline{(\Delta c_1 \Delta c_2)} = 0$ and that $\overline{(\Delta c_1)^2} = \overline{\Delta c_2^2}$ depends only on x we get

$$\frac{\partial P_1}{\partial x} = \frac{\overline{(\Delta c_1)^2}}{2} \left(\frac{\partial^2}{\partial c_1^2} + \frac{\partial^2}{\partial c_2^2} \right) P_1 \,. \tag{4}$$

Changing to variables U and ϑ defined by the equations

$$U = c_1^2 + c_2^2 \,, \qquad \operatorname{tg} \vartheta = \frac{c_2}{c_1} \,, \tag{5}$$

we obtain

$$\frac{\partial \psi}{\partial x} = \overline{\Delta U} \, \frac{\partial}{\partial U} \left(U \frac{\partial \psi}{\partial U} \right) . \tag{6}$$

In (6) $\psi(U, x)$ is the particle distribution function integrated over ϑ. Here the distribution function can depend on ϑ in an arbitrary way; this circumstance will be repeatedly exploited in the following. We solve equation (6) for the boundary condition

$$\psi(U_{\max}, x) = 0 \,, \tag{7}$$

which, although not quite exact, is the condition usually accepted. In (7) $U_{\max}$ is the limiting value of U.

Boundary condition (7) is not a very convenient one as synchrotron oscillation damping leads to a change in $U_{\max}$ during acceleration. We consequently choose as independent the variables y and τ defined by

$$y = \frac{U}{U_{\max}} \,, \qquad \mathrm{d}\tau = \frac{\Delta \bar{U}}{U_{\max}} \, \mathrm{d}x \,. \tag{8}$$

The following equation is valid for $\Xi(y, \tau)$, the particle distribution function in y and τ

$$\frac{\partial \Xi}{\partial \tau} = \frac{\partial}{\partial y} \left(y \frac{\partial \Xi}{\partial y} \right) + \frac{U'_{\max}}{\overline{\Delta U}} \, \frac{\partial}{\partial y} (y \Xi) \,, \tag{9}$$

with the boundary condition

$$\Xi(1, \tau) = 0 \,. \tag{10}$$

The notation $U'_{max} = (d/dx)U_{max}$ has been introduced in (9). We shall compute $U_{max}/\overline{\Delta U}$ on the assumption that U_{max} is determined by the most stable phase of the oscillation. Those cases in which a more rigorous restriction is the maximum momentum deviation (determined by betatron oscillation stability or by the possible amplitude of the radial particle oscillations in the accelerator chamber) are much more rare.

It is not difficult to deduce from (1) and (5) that

$$\frac{U'_{max}}{\overline{\Delta U}} = -\frac{A^2_{max}}{(\widetilde{A^2})'} \cdot \frac{\Omega'}{\Omega} \,, \tag{11}$$

A_{max} is the maximum amplitude of the phase oscillations and $(\widetilde{A^2})' = (d/dx)\widetilde{A^2}$ should be evaluated without taking into account the adiabatic decrease of this quantity resulting from the variation of the synchrotron oscillation frequency.

3. – Solution of the Fokker-Planck equation.

In the general case equation (9) can be solved only by numerical methods, a knowledge of the particular form of the dependence of $U'_{max}/\Delta U$ on τ being of decisive importance.

Calculations of this type should be carried out separately for each given machine. (Some important estimates for this case will be presented in Sect. 6). If however, this quantity is independent of τ an analytic solution necessary for determination of the tolerances can be obtained.

Let us elucidate the meaning of the assumption made above. In the « usual » acceleration region the synchrotron oscillations are damped and $\Omega' < 0$. Let

$$\frac{U'_{max}}{\overline{\Delta U}} = \frac{A^2_{max}}{(\widetilde{A^2})'} \cdot \frac{|\Omega'|}{\Omega} = \varkappa^2 = \text{const} > 0 \,. \tag{12}$$

From (12) it follows immediately that

$$(\widetilde{A^2})' = \frac{1}{\varkappa^2} A^2_{max} \frac{|\Omega'|}{\Omega} \,. \tag{13}$$

Thus the mean square increment of the phase oscillation amplitude is assumed to equal the adiabatid damping of oscillations possessing an amplitude

$A_{max}/\varkappa$. The magnitude of $\varkappa$ must be chosen on the basis of the permissible particle losses.

With account of (12) equation (9) takes the final form

$$\frac{\partial \Xi}{\partial \tau} = \frac{\partial}{\partial y}\left(y \frac{\partial \Xi}{\partial y}\right) + \varkappa^2 \frac{\partial}{\partial y}(y\Xi) . \tag{14}$$

The explicit expression for τ can be found from (8) and (12)

$$\tau = \int_{x_0}^{x} \frac{\overline{\Delta U}}{U_{max}} \, dx = \frac{1}{\varkappa^2} \ln \cdot \frac{\Omega_1}{\Omega} . \tag{15}$$

The general solution of equation (14) can be expressed as follows

$$\Xi = \sum_{S=1}^{\infty} A_S \cdot \left(\frac{\Omega}{\Omega_1}\right)^{|\alpha_S|} \cdot M(\alpha_S, 1, \varkappa^2 y) \cdot \exp[-\varkappa^2 y] , \tag{16}$$

where

$$M(\alpha, 1, \xi) = 1 + \frac{\alpha}{1!\,1!}\xi + \frac{\alpha(\alpha+1)}{2!\,2!}\xi^2 + \frac{\alpha(\alpha+1)(\alpha+2)}{3!\,3!}\xi^3 + \dots ,$$

is the confluent hypergeometric function ([3]). In accord with (10)

$$M(\alpha_S, 1, \varkappa^2) = 0 . \tag{17}$$

Condition (17) defines the set of possible values of the roots α_S of $M(\alpha, 1, \varkappa^2)$ (see Fig. 1). For each term in sum (16) the fraction of remaining particles is determined by the factor $(\Omega/\Omega_1)^{|\alpha_S|}$. As can be seen from Fig. 1 the absolute value of α_S rapidly increases with the index S, so that even $\alpha_2/\alpha_1 \gg 1$. All of the α_S are negative. For small α the following formula is valid

$$M(\alpha, 1, \xi) = 1 + \alpha[\overline{Ei}(\xi) - \ln \xi - 0.577] . \tag{18}$$

Function $\overline{Ei}$ has been tabulated in ref. ([3]). Formula (18) can be applied to find the first roots of the function M and for other calculations as well. To give an idea of the precision of this formula, it might be worthwhile to mention that the errors of the first roots calculated by means of (18) are 18% for $\xi = 2$, 12% for $\xi = 3$ and 7% for $\xi = 4$.

The tolerances associated with the synchrotron oscillation perturbations

([3]) E. JANCKE and F. EMDE: *Tables of Functions* (New York, 1945).

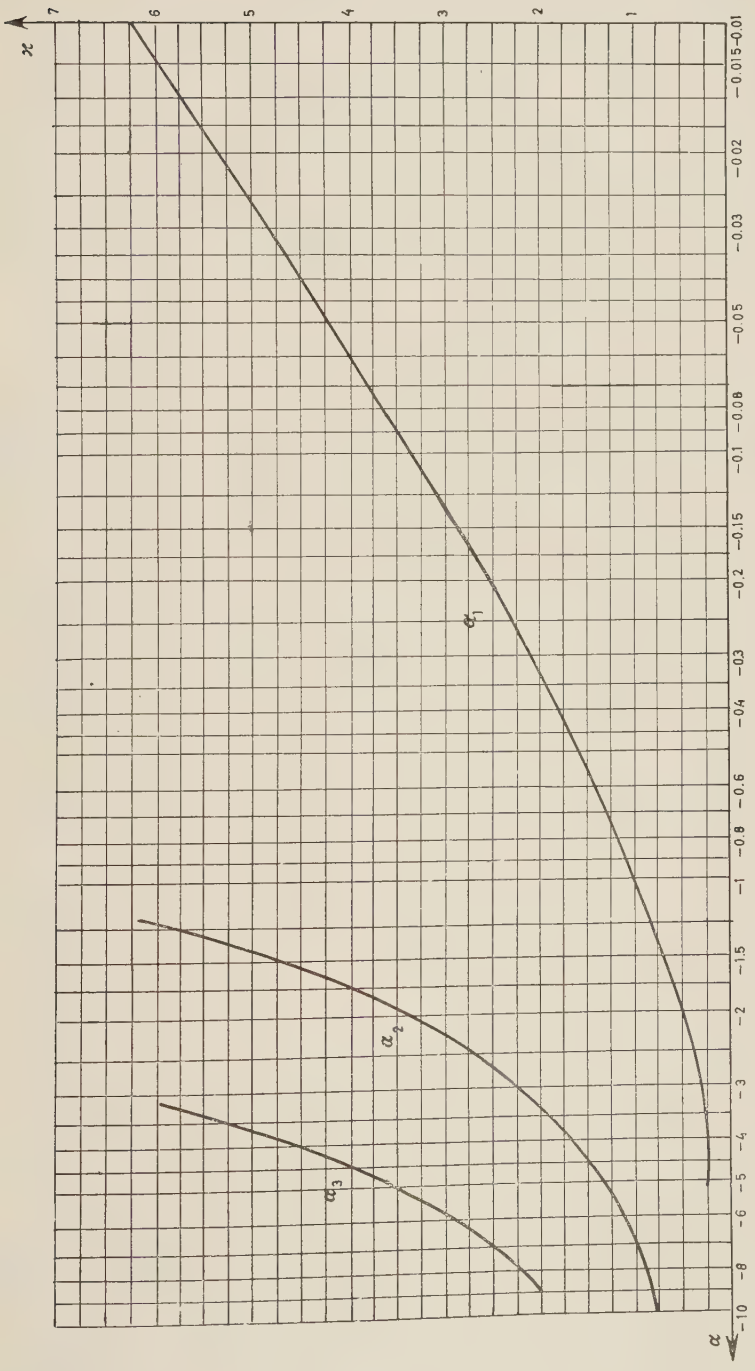

Fig. 1. – Roots of $M(\alpha, 1, \varkappa^2)$ as a function of $\varkappa^2$. α_1 is the first root of the function, α_2 and α_3 are the second and the third roots.

are always quite stringent, and one is thus compelled to consider as inevitable an appreciable particle loss so that

$$K = \left(\frac{\Omega}{\Omega_1}\right)^{|\alpha_1|} \leqslant \frac{1}{2}. \tag{19}$$

In this case all terms in sum (16) except the first should be neglected and hence

$$\Xi \simeq A_1 \left(\frac{\Omega}{\Omega_1}\right)^{|\alpha_1|} \cdot M(\alpha_1, 1, \varkappa^2 y) \exp[-\varkappa^2 y], \tag{20}$$

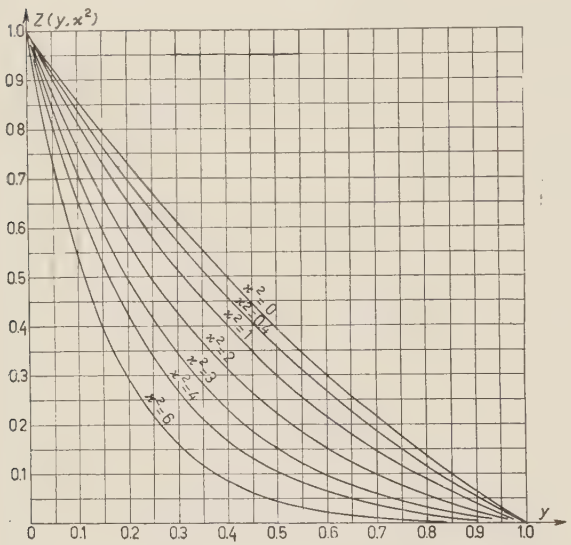

Fig. 2. – Plots of function $Z(y, \varkappa^2) = \exp[-\varkappa^2 y] \cdot M(\alpha_1, 1, \varkappa^2 y)$. For $\varkappa^2 = 0$ this function goes over to $J_0(2.405\sqrt{y})$.

where α_1, is the first root of $M(\alpha, 1, \varkappa^2)$.

Graphs of the first proper function $M(\alpha_1, 1, \varkappa^2 y) \cdot \exp[-\varkappa^2 y]$ are given in Fig. 2.

In practical calculations one should proceed from a given fraction K of particles remaining after acceleration, compute the frequencies at the beginning and the end of acceleration and, using (19), find α_1. Finally $\varkappa^2$ should be found from Fig. 1. For prescribed values of $\varkappa^2$ and $A^2_{\max}$ formula (13) determines the upbuilding rate of the synchrotron oscillations at any given moment and hence the tolerances.

As is well known [1], in strong focusing accelerators, Ω' remains positive ovre a certain period of time after passage through the critical point and therefore $\varkappa^2 < 0$. In this case the solution of equation (14) has the form

$$\Xi = \sum_{S=1}^{\infty} A_S \left(\frac{\Omega_{\text{eff}}}{\Omega}\right)^{1+|\alpha_S|} \cdot M(\alpha_S, 1, \varkappa^2 y). \tag{21}$$

Investigation of the region $\Omega' > 0$ is beyond the scope of the present paper. As was shown by one us [4] if the proper damping is small $|\varkappa^2 \simeq 0|$ the so-

[4] D. G. KOŠKAREV: *Effect of gas scattering on synchrotron oscillations of the accelerated particles* (Pribory i Technika Experimenta, N. 2, 1957).

lution of (14) is

$$\Xi = \sum_{s=1}^{\infty} A_s J_0(\lambda_s \sqrt{\bar{y}}) \cdot \exp\left(-\frac{\lambda_s^2}{4}\tau\right) , \tag{22}$$

where λ_s are the roots of Bessel function J_0.

In analogy with the foregoing it should be concluded that with suitably chosen tolerances all terms of the sum, excluding the first one, may be neglected at the end of the acceleration cycle. The fraction of particles remaining up to the end of the cycle is determined by the quantity

$$\exp\left(-1.45 \int \frac{(\widetilde{A}^2)'}{A_{\max}^2} \mathrm{d}x\right). \tag{23}$$

This formula does not take into account the natural damping of the oscillations and thus yields an underestimate of the amount of remaining particles; however, due to its simpicity it should be quite helpful.

4. – Particle losses at the beginning of the acceleration cycle.

Let $\Xi_0(y_1)$ denote the particle distribution function at injection. The probability that a particle will be accelerated is given by

$$Y_1 = \frac{\int_0^1 \Xi_0(y_1) M(\alpha_1, 1, \varkappa^2 y_1)\, \mathrm{d}y_1 \int_0^1 M(\alpha_1, 1, \varkappa^2 y) \exp[-\varkappa^2 y]\, \mathrm{d}y}{\int_0^1 [M(\alpha_1, 1, \varkappa^2 y)]^2 \exp[-\varkappa^2 y]\, \mathrm{d}y} . \tag{24}$$

(In deriving (24) from (16) it was taken into account that

$$\int_0^1 \exp[-\varkappa^2 y] M(\alpha_m, 1, \varkappa^2 y) M(\alpha_n, 1, \varkappa^2 y) \cdot \mathrm{d}y = \delta_{m,n} \int_0^1 \exp[-\varkappa^2 y][M(\alpha_m, 1, \varkappa^2 y)]^2 \, \mathrm{d}y \Big).$$

Let us consider the function $\Xi_0(y_1)$. The distribution of the particles in the plane Π, φ at the moment of injection is shown in Fig. 3 (Π is the deviation of the particle momentum from the equilibrium value). The size of the phase region along the φ axis depends only on $A_{\max}$ which is the most stable motion amplitude. The maximum momentum deviation $\Pi_{\max}$ is usually determined by inho-

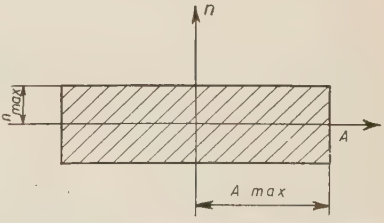

Fig. 3. – Momentum and phase distribution of particles at the instant of injection.

mogeneity of the beam from the injector and is ordinarily not large. Let $\Xi_0(y_1)$ be normalized so that $\int_0^1 \Xi_0(y_1)\,\mathrm{d}y_1 = 1$ and assume that the particle density in the occupied part of the phase plane Π, φ at the injection time is constant; then

$$\Xi_0(y_1) = \frac{E_0\sqrt{U_{\max}}\,\mathrm{ctg}\,\Phi}{2c\Omega_1\Pi_{\max}}\,\arcsin\left[\frac{c\Omega_1\Pi_{\max}\,\mathrm{tg}\,\Phi}{E_0\sqrt{yU_{\max}}}\right]. \tag{25}$$

For arguments larger than 1, the arcsin in (25) should be set equal to $\pi/2$.

Denoting by

$$\frac{\varkappa\cdot c\cdot\Omega_1\Pi_{\max}\,\mathrm{tg}\,\Phi}{E_0 A_{\max}} = D\,,$$

the fraction of captured particles is then

$$Y_1 = \frac{A_{\max}\cdot\varkappa}{2\pi\cdot D}\,\frac{\int_0^{\varkappa^2} M(\alpha_1, 1, \xi)\,\arcsin\left(\frac{D}{\sqrt{\xi}}\right)\mathrm{d}\xi \int_0^{\varkappa^2} M(\alpha_1, 1, \xi)\,\exp[-\xi]\,\mathrm{d}\xi}{\int_0^{\varkappa^2}[M(\alpha_1, 1, \xi)]^2\,\exp[-\xi]\,\mathrm{d}\xi}\,. \tag{26}$$

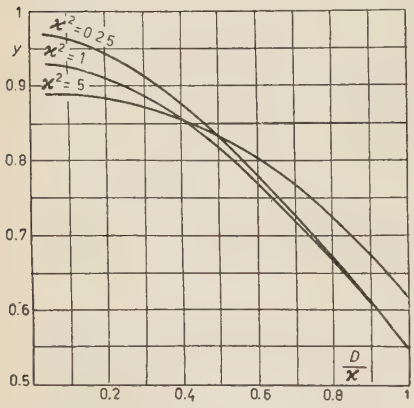

Fig. 4. – Particle losses at the beginning of the acceleration cycle. Graphs of the function $Y=(\pi/A_{\max})Y_1$.

Plots of the function $Y=(\pi/A_{\max})Y_1$ against $\varkappa^2$ for various values of D are shown in Fig. 4.

5. – Tolerance calculations.

A procedure which enables one to find the quantity $(\widetilde{A^2})' = (\mathrm{d}/\mathrm{d}x)\widetilde{A}^2$ for any given particle loss was presented in Sect. 3. As was mentioned above this quantity should be evaluated without taking into account adiabatic damping of the oscillation amplitude.

In the calculations one should first subtract from $(\widetilde{A^2})'$ the quantity $(\widetilde{A_0^2})'$ which is the synchrotron oscillation built up associated with gas scattering (4). The remaining quantity $(\widetilde{A^2})' - (\widetilde{A_0^2})'$ can be split in an arbitrary way into three terms d_1, d_2, d_3 which correspond to noise (and resonance) perturbations of the frequency and amplitude of the accelerating voltage and to the perturbation of the magnetic field strength.

To calculate noise tolerances the results of ref. [1] can be immediately used. In determining the resonance perturbation tolerances one should differentiate between « small resonances » (on passage through which $\Delta U/U_{max} \ll 1$) and rare strong resonance perturbations. The latter will be considered later.

The « small » resonance formulae can most simply be obtained from those for noise perturbations. For a resonance perturbation of amplitude a (in relative units) the mean square value is $\frac{1}{2}a^2$. It can be replaced by a noise perturbation of spectral intensity δ which acts over a period $1/F$ (the distance between neighbouring harmonics is equal to the fundamental frequancy F in cycles per second),

$$\delta = \frac{1}{2F} a^2 . \tag{27}$$

Then

$$\nu + \frac{1}{2F} \cdot (f_\Omega)^2 = \frac{1}{2\pi^2} \cdot \frac{ceu \sin \Phi}{E_0 L} \cdot d_1 , \tag{28}$$

$$\eta + \frac{1}{2F} \cdot \left(\frac{u_\Omega}{u}\right)^2 = \frac{2 \operatorname{ctg}^2 \Phi}{\Omega^2} \cdot \frac{E_0 L}{ceu \sin \Phi} \cdot d_2 , \tag{29}$$

$$\mu + \frac{1}{2F} \cdot \left(\frac{V_\Omega}{V}\right)^2 = \frac{2 \operatorname{ctg}^2 \Phi}{\Omega^2} \cdot \left[\frac{E_0 L}{ceu \sin \Phi}\right]^{-1} \cdot h(\tau_1, x) \cdot d_3 , \tag{30}$$

where ν is the spectral intensity of the noise modulation of the accelerating voltage frequency expressed in cycles^2/cycle, η is the spectral noise intensity of $\Delta u/u$ (the accelerating voltage) in cycle^{-1}, μ is the spectral noise intensity of $\Delta V/V$ (the coil supply voltage) in cycle^{-1}, f_Ω is the amplitude of the radio-frequency ripple (in cycles), u_Ω and V_Ω are respectively the amplitudes of the resonance perturbations of the accelerating voltage and of the voltage applied to the magnet coils; τ_1 is the time constant (in units of x) of the system which automatically couples the frequency of the accelerating voltage with the magnetic field strength.

Function $h(\tau_1, x)$ which enters (30) is defined by the formula

$$h(\tau_1, x) = (1 + \Omega^2 \tau_1^2)^2 \left[\left(1 - \Omega^2 \tau_1^2 \frac{Q}{1 - Q}\right)^2 + \frac{\Omega^2 \tau_1^2}{(1 - Q)^2}\right]^{-1} , \tag{31}$$

$$Q = \alpha(1 + x^2) , \qquad \alpha = \frac{\mathrm{d} \ln L}{\mathrm{d} \ln p} . \tag{32}$$

Instead of μ, one may introduce the spectral noise intensity ε of the mag-

netic field strength $\Delta H/H$. It is not difficult to verify that in this case

$$\varepsilon + \frac{1}{2F}\left(\frac{H_\Omega}{H}\right)^2 = \frac{2\,\mathrm{ctg}^2\,\Phi}{\Omega^4 \cdot x^2}\,\frac{E_0 L}{ceu\,\sin\,\Phi}\,h(\tau_1, x)\cdot d_3\,. \tag{33}$$

Consider now the influence of strong resonances.

The coordinate increments Δc_1 and Δc_2 acquired by the particles after passage through resonance are identical and therefore some of the particles will be forced out of the region of stable motion and will be instantaneously lost.

The fraction of instantaneously lost particles, $(1 - B_1)$ is

$$1 - B_1 = \frac{\int_0^{\varkappa^2} M\,(\alpha_1, 1, \xi)\,\exp\,[-\xi]\,\arccos\left(\dfrac{\varkappa^2 - \xi - \varkappa^2 \Delta^2}{2\varkappa\Delta\sqrt{\xi}}\right)\cdot \mathrm{d}\xi}{\int_0^{\varkappa^2} M(\alpha_1, 1, \xi)\,\exp\,[-\xi]\,\mathrm{d}\xi}\,, \tag{34}$$

where

$$g = \varkappa^2(1 - \Delta)\,, \qquad 0 \leqslant \Delta \leqslant 1\,; \qquad g = 0\,, \qquad 1 \leqslant \Delta \leqslant 2\,.$$

In formula (34) arc cos should be set equal to π if the argument is $\leqslant -1$. The symbol Δ denotes the « displacement » in the c_1, c_2 plane. The fraction of remaining particles, B_1, is shown in Fig. 5. This, however, is not the only mode of particle losses. As shown above, the higher terms in expansion (16) rapidly decrease and thus we should be consider as finally remaining only that fraction B_2 of the particles which corresponds to the first term of the proper

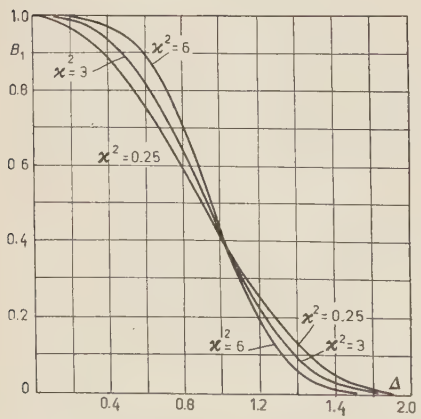

Fig. 5. – Instantaneous particle loss due to resonance perturbations. (The fraction of remaining particles is plotted in the graph).

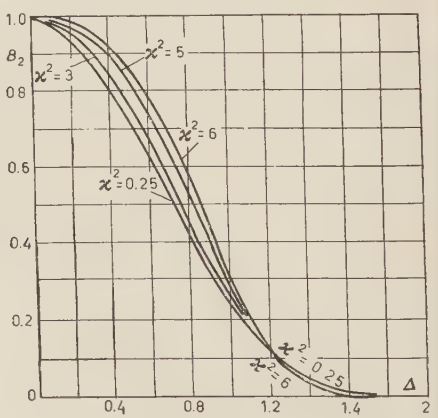

Fig. 6. – Total particle loss due to resonance perturbations (The fraction of remaining particles is plotted in the graph).

function expansion of the new post-resonance particle distribution. B_2 is defined by the expression

$$B_2 = \frac{1}{2\pi} \frac{\iint M(\alpha_1, 1, \xi) M(\alpha_1, 1, \zeta) \exp[-\zeta] \, \mathrm{d}\xi \, \mathrm{d}\zeta}{\int_0^{\varkappa^2} [M(\alpha_1, 1, \xi)]^2 \exp[-\xi] \, \mathrm{d}\xi}, \tag{35}$$

$$\zeta = \xi + \varkappa^2 \Delta^2 - 2\varkappa\Delta\sqrt{\xi} \cos\vartheta .$$

Integration in the numerator is carried out over the region $0 \leqslant \xi \leqslant \varkappa^2$, $0 \leqslant \zeta \leqslant \varkappa^2$. A plot of B_2 is presented in Fig. 6.

The choice between formulae (34) and (35) depends on how far from the end of the acceleration cycle the resonance perturbation occurred: if it took place at the end of the acceleration, formula (34) should be used; if, on the contrary, sufficient time elapsed after passage through resonance, all higher proper functions of the system die out, and formula (35) should be applied.

6. – Particle losses for arbitrary perturbations.

Up to this stage the constancy of $\varkappa^2$ during a complete acceleration cycle was important for the development of the theory. Equation (14) does not have an analytic solution for arbitrary perturbations.

However, if we neglect losses associated with variation of the proper functions (which, as can be seen from Fig. 2, only slightly differ from each other) the particle loss formula can readily be generalized for the case of an arbitrary dependence of $\varkappa^2$ on x. The fraction of remaining particles K is then given by the expression

$$K = \exp\left[-\int_{x_0}^{x} |\alpha_1| \frac{\Omega'}{\Omega} \, \mathrm{d}x\right]. \tag{36}$$

In (3), $|\alpha_1|$ depends on x through $\varkappa^2$ in accord with (13). A plot of $\alpha_1(\varkappa)^2$ is shown in Fig. 1.

Empirical formulae can be chosen for the function $\alpha_1(\varkappa^2)$. For $\varkappa^2 \geqslant 2$ it may be considered that

$$|\alpha_1| = 1.56 \exp[-0.81\,\varkappa^2] . \tag{37}$$

For $\varkappa^2 \leqslant 1.5$ good accuracy can be attained by using the formula

$$|\alpha_1|\varkappa^2 = 1.45 - 0.45\varkappa^2 . \tag{38}$$

In those cases in which (38) is valid, formula (36) acquires the simple form

$$K = \left(\frac{\Omega_1}{\Omega}\right)^{0,45} \cdot \exp\left[-1.45 \int_{x_0}^{x} \frac{(\widetilde{A^2})'}{A_{\max}^2}\, \mathrm{d}x\right]. \tag{39}$$

The values given by formulae (36) and (39) for the number of remaining particles are somewhat too large. It is interesting to compare (39) with (23) which, as mentioned in its deduction, always yields a value for the number of remaining particles which is somewhat too low. These two formulae are very similar and differ only by the term $(\Omega_1/\Omega)^{0,45}$ which accounts for adiabatic damping of the oscillations.

* * *

The authors are grateful to Prof. V. V. VLADIMIRSKIJ for discussion of this work, to the Laboratory of Computing Machines and its Director I. S. BRUK, Corresponding Member of the Academy of Sciences of the USSR, for kind persmission to carry out the necessary calculations on Electronic M-2 and to N. M. SUHAČOVA for performing the calculations.

RIASSUNTO (*)

Si esaminano vari tipi di perturbazioni delle oscillazioni di sincrotrone di particelle accelerate dal punto di vista delle perdite di particelle che producono. Si stabilisce un sistema di tolleranze per una prefissata perdita di particelle.

(*) *Traduzione a cura della Redazione.*

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The Orbital Electron Capture Decay of ^{185}Os.

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(ricevuto il 7 Maggio 1957)

Summary. — Radiations emitted in the orbital electron capture decay of ^{185}Os have been investigated by means of a scintillation spectrometer and with the help of coincidence methods. The total decay energy was found to be 984 keV: the orbital electron capture decay is complex and one of the transitions involved shows an anomalous L/K-capture ratio. It is suggested that this transition can be classified as second forbidden ($\Delta I = 2$; yes).

1. – Introduction.

^{185}Os is known to decay with a half life of 97 days to ^{185}Re. No positron emission was observed; conversely intense γ-rays having the energies 645 keV and 879 keV were found by several authors ([1-4]) together with some γ-rays of much lower intensities. These facts show that the total decay energy in the electron capture decay of ^{185}Os must be greater than 879 keV and less than 1.02 MeV. The L/K-capture ratio was measured by Miller and Wilkinson ([2]) by comparing the intensities of KX and LX radiations from ^{185}Re and found to be 0.35 ± 0.15; owing to the complexity of the considered electron capture transition, the uncertainty of evaluating counting efficiencies for soft radiations as LX radiations and to the lack of exact knowledge of

([1]) J. M. Hollander, I. Perlman and G. T. Seaborg: *Rev. Mod. Phys.*, **25**, 469 (1953).

([2]) M. M. Miller and R. G. Wilkinson: *Phys. Rev.*, **83**, 1050 (1951).

([3]) J. M. Cork, J. M. Le Blanc, W. H. Nester, D. W. Martin and K. M. Brice: *Phys. Rev.*, **90**, 444 (1953).

([4]) C. H. Pruett and R. G. Wilkinson: *Phys. Rev.*, **100**, 1237 (1955).

L series fluorescence yields, this value cannot be considered very accurate. Therefore a further examination of the radiation emitted from ^{185}Os and a new measurement of the L/K^- capture ratio by means of coincidence techniques [5] were thought to be useful.

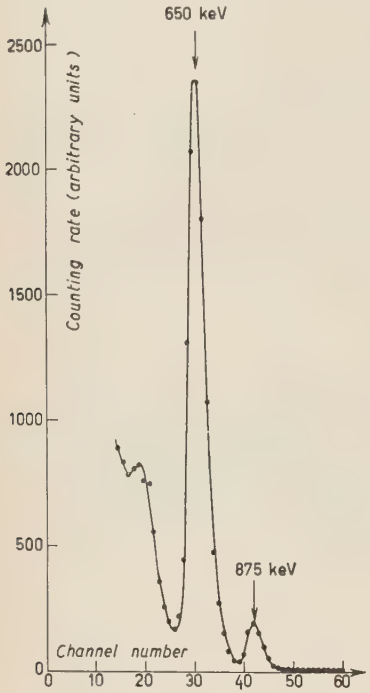

Fig. 1. – High energy γ spectrum from ^{185}Os.

2. – Experimental results.

Active Os was obtained by slow neutron irradiation and supplied by A.E.R.E. (Harwell). One year after irradiation the only important activity is due to ^{185}Os.

1) A spectrum of high energy γ-rays, as obtained with a source located at 2.5 cm from a single crystal γ spectrometer, is shown in Fig. 1. The well-known lines of 650 keV and 875 keV are clearly visible; their intensity ratio resulted to be 7.6 when efficiencies were taken into account according to the data given by BELL [6]. No other γ lines were apparent in the spectrum.

The only important peaks in the low energy region were attributable to characteristic KX- and LX-rays of Re (60 keV and 9.5 keV respectively). LX-rays were observed by means of a A+CH_4 filled proportional counter. The line due to KX-rays is shown in Fig. 2: it is clearly asymmetrical at the right side and this fact was interpreted as due to the presence of a γ line at about 70 keV.

TABLE I.

γ or X line	Relative intensity
875 keV	0.188
650 keV	1.435
KX-rays	1.00
LX-rays	1.6

[5] A. BISI, E. GERMAGNOLI and L. ZAPPA: *Nuclear Physics*, **1**, 593 (1956).

[6] P. R. BELL in K. SIEGBAHN: *Beta- and Gamma-Ray Spectroscopy* (Amsterdam, 1955), p. 133.

Table I gives the relative intensities of X and γ transitions: fluorescence yields for X radiations have been taken into account.

2) Both the above mentioned γ lines and the KX peak were in turn used to trigger a coincidence circuit (resolving time about 2 μs). In this way possible γ-γ cascades and γ-X coincidences were searched for information about the decay scheme.

Only X-rays were found to be emitted in coincidence with the 875 keV γ line; two weak, only partially resolved, γ lines at about 160 and 230 keV, and X-rays were distinguished in the coincidence spectrum triggered by the 650 keV line. Finally the whole γ spectrum (160, 230, 650 and 875 keV lines) resulted to be coincident with KX-rays. Table II summarizes the results for coincidence measurements: efficiencies and fluorescence yields were taken into account and in any case no attempt was made to distinguish between KX-rays and possible 70 keV γ-rays.

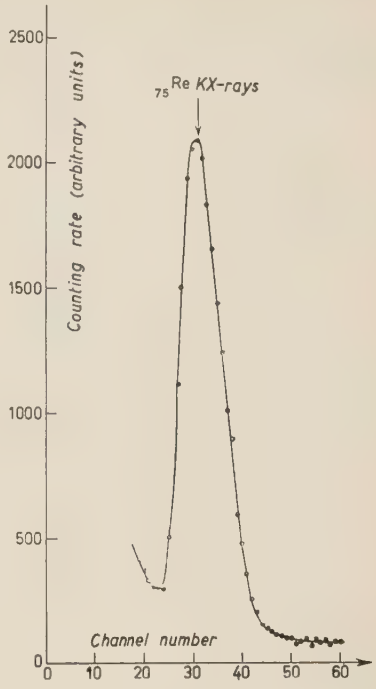

Fig. 2. – KX-ray line of $_{75}$Re.

Consequently the decay scheme given in Fig. 3 was deduced. Intensity measurements from the γ spectrum and coincidences support such a scheme.

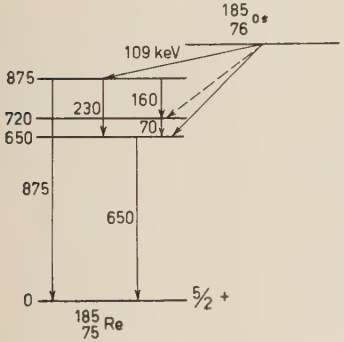

Fig. 3. – Decay scheme of ^{185}Os.

3) From the coincidence measurements with the 875 keV line used as triggering peak, the L/K capture ratio for the orbital electron capture transition between ^{185}Os and the 875 keV level of ^{185}Re can be deduced according to the formula

$$\frac{I_{875,K}}{I_{875}} = \varepsilon_K \omega_K \frac{P_K}{P_L + P_K}, \tag{1}$$

where $(1/\varepsilon_K\omega_K)(I_{875.K}/I_{875})$ results from Table II.

Here ε_K is the counter efficiency for KX-rays (escape peak corrected), ω_K is the fluorescence yield for K-shell in Re ($\omega_K = 0.95$); P^L and P_K are the electron capture probabilities for L- and K-shell. The result

$$\frac{P_L}{P_K} = 1.08 \pm 0.04,$$

is readily obtained.

TABLE II.

Triggering peak	Coincidence intensity/intensity of triggering peak				
	160 keV	230 keV	650 keV	875 keV	KX-rays
875 keV	—	—	—	—	0.48
650 keV	weak	weak	—	—	0.63
KX-rays	weak	weak	0.86	0.095	0.06

4) No definite conclusions can be drawn from similar measurements involving the 650 keV γ line. Actually KX-rays are found to be coincident with such a line, but they can arise both from electron capture in the transition between ^{185}Os and the level at 650 keV in ^{185}Re (and perhaps a level at about 720 keV) and from internal conversions of the 230 keV and 160 keV γ lines. Moreover a contribution to the coincidence rate from unconverted γ-rays of 70 keV cannot be excluded. Anyway, in the extreme hypothesis that all the low energy γ transitions are of negligible intensity, we find, in strict analogy with (1):

$$\left(\frac{P_L}{P_K}\right) = 0.59 \pm 0.04 ,$$

This value has to be considered as an upper limit for the L/K ratio for the orbital electron capture transition between ^{185}Os and the 650 keV level of ^{185}Re.

3. – Discussion.

Although the above decay scheme, which substantially agrees with the conclusions of previous authors, is likely to be correct, only few quantitative statements can be drawn from the present measurements, as far as concerns the relative probabilities of the orbital electron capture transitions to the different levels of ^{185}Re and the multipolarities of the same levels.

With any reasonable assumption about the multiple order of the 875 keV γ transition, the corresponding γ line is expected to be weakly internally converted. Moreover the 230 keV and 160 keV transitions can for a first approximation be neglected. Consequently the electron capture process towards the 875 keV level occurs with about 12% probability. The P_L/P_K ratio is equal to 1.08 ± 0.04 according to (1): for an allowed transition ($\Delta I = 0, 1$; no) an energy of (109 ± 2) keV can be deduced according to the formula given by ROSE and JACKSON (7). A result which is not quite different from the present

(7) M. E. ROSE and J. L. JACKSON: *Phys. Rev.*, **76**, 1540 (1949).

one can be obtained if the transition is assumed to have first forbidden character ($\Delta I = 0, 1$; yes). The estimation of the value of $\log ft$ ($\log ft \sim 6.6$) prevents any other hypothesis; the total decay energy of ^{185}Os is consequently calculated to be 984 keV.

A comparison between the intensities of 650 keV γ rays and KX-rays, as given by Table I, shows that the transition between the ground levels of ^{185}Os and ^{185}Re has a small probability, if any. Therefore most KX-rays are emitted in decays by electron capture processes from ^{185}Os to the level of ^{185}Re at 650 keV, or to the level at about 720 keV, or to both. This agrees with what has been found in KX-650 keV coincidence measurements. In these two cases the transition energies must be about 330 and 260 keV; values for the P_L/P_K ratios for such transitions can be easily calculated to be 0.20 or 0.25 for allowed transitions. The experimentally found value is 0.59 ± 0.04, which has to be interpreted as a weighted average for the two transitions and is definitely larger than the expected ones.

If the possibility that the γ transitions of 230 keV and 160 keV are not negligibly weak is taken into account and also a contribution to coincidence rate from unconverted 70 keV γ-rays is allowed for, the preceding conclusions are even more valid. One is therefore forced to assume that either one of the considered electron capture processes, or both, are at least second forbidden ($\Delta I = 2$; yes) because only in this case the P_L/P_K value can be significantly higher that the expected one for allowed transiton ([8]). Also the high intensity of LX-rays emitted in the decay of ^{185}Os and the small rate of KX-KX coincidences support these conclusions.

Owing to the complexity of the orbital electron capture decay of ^{185}Os and to the uncertainty in spin and parity of the ground level of ^{185}Os it is hard to make unique assignments of orbitals of the excited levels of ^{185}Re: no serious attempts have been made to this end.

* * *

We wish to thank Prof. G. Bolla for his interest in this work.

([3]) H. Brysk and M. E. Rose: *ORNL* 1830 (1955).

RIASSUNTO

Le radiazioni emesse nel decadimento per cattura elettronica orbitale dell'^{185}Os sono state studiate per mezzo di uno spettrometro a scintillazione e con metodi di coincidenze. L'energia totale del decadimento è risultata uguale a 984 keV; una delle transizioni di cattura elettronica orbitale interessate nel decadimento è probabilmente proibita dal secondo ordine ($\Delta I = 2$; sì), come suggerito dal valore anomalo del rapporto tra le probabilità di cattura nelle orbite L e K.

IL NUOVO CIMENTO Vol. VI, N. 2 1° Agosto 1957

Individual Mass Reversal and Parity Non-Conservation.

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(ricevuto il 7 Maggio 1957)

Summary. — Some consequences of the assumption that every interaction which exists in Nature never allows us to determine the signs of masses of all elementary particles, are discussed. The assumption entails the postulate of invariance of all interactions under a set of transformations which we call Individual Mass Reversal. The electromagnetic interaction and the π-N interaction, being parity-conserving, are compatible with the postulate, provided that the latter is a PS(PV)-coupling. Most of Fermi interactions and interactions such as $\Lambda N\pi$ and ΛNK are not allowed, unless they do not conserve parity. Although some strong interactions are parity-non-conserving in our theory, no serious difficulty does arise if we redefine a charge conjugate state as was suggested by L. Landau, since our theory is proved to be invariant under the product of charge conjugation and space inversion. Our theory is equivalent to the two component neutrino theory of T. D. Lee and C. N. Yang for neutrino processes, except the μ-e decay.

1. – Introduction.

There are several experimental methods at present to measure the mass of an elementary particle. Each of them, however, measures not the mass itself but the rest energy (in the unit of $c = 1$) $\lim_{p\to 0}\sqrt{p^2+m^2}$ of the particle, that is, the absolute value of the mass. No attempt to determine the sign of a mass has succeeded. We generalize the fact and assume that every interaction which exists in Nature never allows us to determine the signs of masses of all elementary particles. The assumption entails a postulate of invariance of all interactions under a set of transformations, to which we shall refer as Individual Mass Reversal (IMR). This postulate is valid, only if parity does not conserve. The question of parity non-conser vation for weak

interactions was extensively investigated by T. D. LEE and C. N. YANG (1) and T. D. LEE, R. OEHME and C. N. YANG (2). Experimental verification of parity non-conservation have been given by C. S. WU *et al.* (3) for a β-decay interaction and by R. L. GARWIN *et al.* (4) for the π-μ and μ-e decays.

2. – The definition of individual mass reversal.

We shall distinguish particles by letters a, b, c, ... and denote particles pertaining to a same charge multiplet state by a same letter in order to retain the charge independence and denote also their antiparticles by the same letter in order for the operation of the transformation to commute with the charge conjugation operation. The IMR of a spin $\frac{1}{2}$ particle a, M_a, is defined as follows:

$$m_a \to -m_a\,, \qquad m_b \to m_b\,, \qquad m_c \to m_c\,, \qquad m_d \to m_d\,, \quad \ldots$$

$$\psi_a \to \eta^a{}_a \gamma_5 \psi_a\,, \qquad J_b \to \eta^a{}_b \psi_b\,,$$

$$\varphi_c \to \eta^a{}_c \varphi_c\,, \qquad \hat{\varphi}_d \to \eta^a{}_d \varphi_d\,,$$

where ψ_b represents a spin $\frac{1}{2}$ particle other than a and may have arbitrary parity and φ_c, $\hat{\varphi}_d$ denote a scalar and a pseudoscalar wave function respectively, η's being phase factors to be determined. A vector or axial vector wave function (φ_μ or $\hat{\varphi}_\mu$) transforms like $\partial_\mu \varphi_c$ or $\partial_\mu \hat{\varphi}_d$ respectively under the IMR as well as the space inversion P. Every interaction should be invariant under the whole set of the transformations M_a, M_b, ...

The IMR defined here should not be confused with the simple mass reversal which was treated, for instance, by J. TIOMNO (5) or T. OUCHI, K. SEMBA and M. YONEZAWA (6). The latter does not require parity-non-conservation.

3. – Restrictions to interactions.

At first we examine interactions of the form $\overline{\psi}_b O \psi_b \varphi$. The interactions will be invariant under M_a if we choose $\eta^a{}_c = \eta^a{}_d = 1$. Thus boson fields should be unaffected by the IMR operation. Interactions of the form $\overline{\psi}_a O \psi_a \varphi$ can

(1) T. D. LEE and C. N. YANG: *Phys. Rev.*, **104**, 254 (1956).

(2) T. D. LEE, R. OEHME and C. N. YANG: to be published.

(3) C. S. WU, E. AMBLER, R. W. HAYWARD, D. D. HOPPES and R. P. HUDSON: *Phys. Rev.*, **105**, 1413 (1957).

(4) R. L. GARWIN, L. M. LEDERMAN and M. WEINRICH: *Phys. Rev.*, **105**, 1415 (1957).

(5) J. TIOMNO: *Nuovo Cimento*, **1**, 226 (1955). He considered IMR of a particle which interacts only with the electromagnetic field.

(6) T. OUCHI, K. SEMBA and M. YONEZAWA: *Prog. Theor. Phys.*, **15**, 431 (1956).

be made to satisfy the parity conservation. It is through these interactions that we can define the parity of a particle.

The interactions $s(S)$, $ps(Ps)$, $s(V)$, $v(T)$ and $pv(T)$ should be, however, given up, because they change their sign under M_a. The only allowed interactions which are invariant under both P and M_a, are $s(V)$, $ps(Pv)$, $v(V)$ and $pv(Pv)$:

$$\bar{\psi}_a\gamma_\mu\psi_a\left(\frac{\partial\varphi}{\partial x_\mu}\text{ or }\varphi_\mu\right) \quad \text{and} \quad \bar{\psi}_a\gamma_5\gamma_\mu\psi_a\left(\frac{\partial\hat{\varphi}}{\partial x_\mu}\text{ or }\hat{\varphi}_\mu\right).$$

Thus both the electromagnetic interaction and the π-N interaction are parity-conserving and allowed by our criterion, if the latter is $ps(Pv)$-coupling. (We do not think the renormalizability to be a sensible criterion for the future theory). An argument which is favorable to derivative couplings was given by K. IWATA *et al.* ([7]).

The interactions of the form $\bar{\psi}_aO\psi_b\varphi$ $(a\neq b)$ cannot be parity-conserving. That kind of interactions which are invariant under the transformation M_a, are confined to

$$\begin{aligned} &S \pm P: && \bar{\psi}_a(1\pm\gamma_5)\psi_b\,, \\ &V \mp A: && \bar{\psi}_a(1\mp\gamma_5)\gamma_\mu\psi_b\,, \\ &T \pm \tilde{T}: && \bar{\psi}_a(1\pm\gamma_5)\sigma_{\mu\nu}\psi_b\,, \qquad (\sim\text{: dual}) \end{aligned}$$

where we ignored boson parts since they are already proved to be invariant under the IMR.

Thus IMR-invariant interactions of the form are,

(I) $\quad g\bar{\psi}_a(1-\gamma_5)\psi_b(\varphi \text{ or } \hat{\varphi}) + \text{H.C.}\,, \quad g\bar{\psi}_a(1-\gamma_5)\sigma_{\mu\nu}\psi_b(\partial_\mu\varphi_\nu \text{ or } \partial_\mu\hat{\varphi}_\nu) + \text{H.C.}$

if $\eta^{a*}_a\eta^a_b = 1$ and $\eta^{b*}_b\eta^b_a = -1$

(for example if $\eta^a_a = \eta^b_b = \eta^a_b = -\eta^b_a = 1$)

and

(II) $\quad g\bar{\psi}_a(1-\gamma_5)\gamma_\mu\psi_b(\partial_\mu\varphi,\ \partial_\mu\hat{\varphi},\ \varphi_\mu \text{ or } \hat{\varphi}_\mu) + \text{H.C.}$

if $\eta^{a*}_a\eta^a_b = \eta^{b*}_b\eta^b_a = 1$

(for example if $\eta^a_a = \eta^b_b = \eta^a_b = \eta^b_a = 1$)

([7]) K. IWATA, S. OGAWA, H. OKONOGI, B. SAKITA and S. ONEDA: *Prog. Theor. Phys.*, **13**, 19 (1955).

and

(III) $g\overline{\psi}_a(1+\gamma_5)\psi_b(\varphi \text{ or } \hat{\varphi}) + \text{H.C.}, \quad g\overline{\psi}_a(1+\gamma_5)\sigma_{\mu\nu}\psi_b(\partial_\mu\varphi_\nu \text{ or } \partial_\mu\hat{\varphi}_\nu) + \text{H.C.},$

if $\eta^{a*}_a\eta^a_b = -1, \ \eta^{b*}_b\eta^b_a = 1$

(for example if $\eta^a_a = \eta^b_b = -\eta^a_b = \eta^b_a = 1$)

and

(IV) $$g\overline{\psi}_a(1+\gamma_5)\gamma_\mu\psi_b(\partial_\mu\varphi, \ \partial_\mu\hat{\varphi}, \ \varphi_\mu \text{ or } \hat{\varphi}_\mu) + \text{H.C.},$$

if $\eta^{a*}_a\eta^a_b = \eta^{b*}_b\eta^b_a = -1$

(for example if $\eta^a_a = \eta^b_b = -\eta^a_b = -\eta^b_a = 1$).

Only one of these four cases is realized in Nature.

Also a Fermi interaction does not in general conserve parity in our theory. When four participating particles are different from each other, the allowed forms of Fermi interactions are classified corresponding to the above four cases:

(I) $$\begin{cases} f\overline{\psi}_a(1-\gamma_5)\psi_b\overline{\psi}_c(1-\gamma_5)\psi_d + \text{H.C.} & \text{and} \\ f\overline{\psi}_a(1-\gamma_5)\sigma_{\mu\nu}\psi_b\overline{\psi}_c(1-\gamma_5)\sigma_{\mu\nu}\psi_d + \text{H. C.}, \end{cases}$$

(II) $$f\overline{\psi}_a(1-\gamma_5)\gamma_\mu\psi_b\overline{\psi}_c(1-\gamma_5)\gamma_\mu\psi_d + \text{H.C.},$$

(III) $$\begin{cases} f\overline{\psi}_a(1+\gamma_5)\psi_b\overline{\psi}_c(1+\gamma_5)\psi_d + \text{H.C.} & \text{and} \\ f\overline{\psi}_a(1+\gamma_5)\sigma_{\mu\nu}\psi_b\overline{\psi}_c(1+\gamma_5)\sigma_{\mu\nu}\psi_d + \text{H.C.}, \end{cases}$$

(IV) $$f\overline{\psi}_a(1+\gamma_5)\gamma_\mu\psi_b\overline{\psi}_c(1+\gamma_5)\gamma_\mu\psi_d + \text{H.C.}$$

and when two particles, say a and b, are identical (the β-decay interaction etc.) they are

(I) $$\begin{cases} f\overline{\psi}_a\psi_a\overline{\psi}_c(1-\gamma_5)\psi_d + \text{H.C.}, & f\overline{\psi}_a\gamma_5\psi_a\overline{\psi}_c(1-\gamma_5)\psi_d + \text{H.C.}, \\ f\overline{\psi}_a\sigma_{\mu\nu}\psi_a\overline{\psi}_c(1-\gamma_5)\sigma_{\mu\nu}\psi_d + \text{H.C.}, & f\overline{\psi}_a\gamma_5\sigma_{\mu\nu}\psi_a\overline{\psi}_c(1-\gamma_5)\sigma_{\mu\nu}\psi_d + \text{H.C.}, \end{cases}$$

(II) $$\begin{cases} f\overline{\psi}_a\gamma_\mu\psi_a\overline{\psi}_c(1-\gamma_5)\delta\gamma_\mu\psi_d + \text{H.C.}, \\ f\overline{\psi}_a\gamma_5\gamma_\mu\psi_a\overline{\psi}_c(1-\gamma_5)\gamma_\mu\psi_d \quad \text{H.C.} \quad , \end{cases}$$

$$\text{(III)} \quad \begin{cases} f\bar{\psi}_a\psi_a\bar{\psi}_c(1+\gamma_5)\psi_d + \text{H.C.}, & f\bar{\psi}_a\gamma_5\psi_a\bar{\psi}_c(1+\gamma_5)\psi_d + \text{H.C.}, \\ f\bar{\psi}_a\sigma_{\mu\nu}\psi_a\bar{\psi}_c(1+\gamma_5)\sigma_{\mu\nu}\psi_a + \text{H.C.}, & f\bar{\psi}_a\gamma_5\sigma_{\mu\nu}\psi_a\bar{\psi}_c(1+\gamma_5)\sigma_{\mu\nu}\psi_d + \text{H.}, \end{cases}$$

$$\text{(IV)} \quad f\bar{\psi}_a\gamma_\mu\psi_a\bar{\psi}_c(1+\gamma_5)\gamma_\mu\psi_d + \text{H.C.}, \quad f\bar{\psi}_a\gamma_5\gamma_\mu\psi_a\bar{\psi}_c(1+\gamma_5)\psi_d + \text{H.C.C.}.$$

4. – Parity non-conserving strong interactions.

Parity non-conserving interactions are not necessarily weak in our theory. Some strong interactions (ΛNK, etc.) do not conserve parity. Nevertheless we could find no difficulty in our theory, nothing inconsistent with the present experimental data. Our theory turns out to be invariant under the combined inversion (8) which is a product of charge conjugation C (or more specifically speaking, particle-antiparticle conjugation) and space inversion P. Therefore our theory does not yield a large dipole moment to a particle. Next, it seems difficult at first sight to explain equality of the lives of K^+ and K^- particles (and similarly π^+ and π^- or μ^+ and μ^-). A method to avoid the difficulty is, as was suggested by L. LANDAU (9), to redefine an antiparticle state. If an antiparticle is not merely the charge conjugate state of a particle but the charge conjugate and reflected state, the lives of K^+ and K^- particles coincide automatically.

5. – Neutrino processes.

It can be easily seen that our theory is equivalent to the two component neutrino theory of T. D. LEE and C. N. YANG (10) for the β-decay and π-μ decay interactions. The exceptional case is the μ-e decay interaction, in which two neutrinos appear simultaneously. The interaction takes the form,

$$\mathcal{H}_{\mu e} = \sum_i g_i(\bar{\mu}(1 \pm \gamma_5)O^i e)(\bar{\nu}O^i\nu) ,$$

in our theory, where μ, e and ν stand for wave functions of a μ-meson, an electron and a neutrino respectively and $\bar{\mu} = \mu^+\gamma_4$, etc. The suffix i distinguishes between the five types of a Fermi interaction and the coupling con-

(8) W. PAULI: *Niels Bohr and the Development of Physics* (London, 1955). See also reference (2). The term *combined inversion* is due to L. LANDAU (9).

(9) L. LANDAU: preprint.

(10) T. D. LEE and C. N. YANG (to be published). An example of its application was given by S. ONEDA. The authors are thankful to him for his sending them his paper before publishing.

stants g_i are not arbitrary but are restricted by

$$g_v = g_A = 0 \qquad \text{in the case I or III},$$
$$g_s = g_T = g_p = 0 \qquad \text{in the case II or IV}.$$

O^i denotes 1, γ_μ, $\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, $i\gamma_5\gamma_\mu$ and γ_5 for $i = S$, V, T, A and P respectively.

We make use of the relation [11],

$$F_{\alpha\beta}G_{\varrho\sigma} = \tau \sum_i O^i_{\varrho\beta}(FO^iG)_{\alpha\sigma}$$

twice, which holds for arbitrary Dirac matrices F and G. Then we have

$$\mathcal{H}_{\mu e} = \sum_{ijk} g_i a^{ij} a^{jk} (\bar{\mu} O^k e)(\bar{\nu} O^k (1 \pm \varepsilon_j \gamma_5)\nu) ,$$

where

$$a^{ij} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 1 & -\frac{1}{2} & 0 & \frac{1}{2} & -1 \\ \frac{3}{2} & 0 & -\frac{1}{2} & 0 & \frac{3}{2} \\ 1 & \frac{1}{2} & 0 & -\frac{1}{2} & -1 \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \quad (12)$$

and

$$\varepsilon_j = 1 \qquad \text{for} \qquad j = S,\ T,\ P,$$
$$\varepsilon_j = -1 \qquad \text{for} \qquad j = V,\ A .$$

The above expression can be simplified further, if we use

$$\sum_j a^{ij} a^{jk} = \delta_{ik} , \qquad \sum_j a_{ij}\varepsilon_j a^{jk} = \delta_{6-i,k} ,$$

where $i = 1, 2, ..., 5$ correspond to S, V, ..., P. We obtain

$$\mathcal{H}_{\mu e} = (\bar{\mu} e)(\bar{\nu}(g_S \pm g_P\gamma_5)\nu) + (\bar{\mu}\gamma_\mu e)(\bar{\nu}\gamma_\mu(g_V \pm g_A\gamma_5)\nu) +$$
$$+ g_T \sum_{\mu<\nu} (\bar{\mu}\sigma_{\mu\nu} e)(\bar{\nu}\sigma_{\mu\nu}(1 \pm \gamma_5)\nu) - (\bar{\mu}\gamma_5\gamma_\mu e)(\bar{\nu}\gamma_5\gamma_\mu(g_A \pm g_V\gamma_5)\nu) +$$
$$+ (\bar{\mu}\gamma_5 e)(\bar{\nu}\gamma_5(g_P \pm g_S\gamma_5)\nu) .$$

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(11) See for instance H. UMEZAWA: *Quantum Field Theory* (Amsterdam, 1956).

(12) M. FIERZ: *Zeits. f. Phys.*, **104**, 553 (1937); L. MICHEL: *Proc. Phys. Soc.*, A **63**, 514 (1950); C. J. MCCALLUM and A. S. WHIGTMAN: *Technical Report* No. 7; See also H. UMEZAWA: *Quantum Field Theory* No. 7, Ch. VII, § 3. The coefficient a^{ij} comes from the definition $\frac{1}{4} O^i O^j O^i O = a^{ij} O^j$, where the summation over the components within a type i is implicitly understood on the left hand side of the equation.

Thus our μ-e decay interaction $\mathcal{H}_{\mu e}$ is equivalent to that in the two component neutrino theory, only if

$$g_P = \pm g_S , \qquad g_A = \mp g_V .$$

* * *

The authors should like to express their sincere thanks to Prof. T. TATI for his valuable discussions.

Note added in Proof.

If we assume that a hyperon is an excited state of a nucleon (J. SCHWINGER: *Phys. Rev.*, **104**, 1164 (1956)) and masses of all baryons simultaneously change their signs under the IMR. Then every strong interaction becomes parity-conserving. The τ-θ puzzle is left unsolved.

K. IWATA and K. FUJII (Hokkaido University, Sapporo, Japan) suggested a method to get over the difficulty, applying our theory to the Sakata Model (*Prog. Theor. Phys.*, **16**, 686 (1956)), in which all elementary interactions are assumed to be only Fermi interactions and the only particles which undergo strong interactions are Λ and N. They assumed a Fermi interaction $f(\bar{a}O^i b)(\bar{c}O^i d)$ is strong ($f_s \simeq 10^{-42}$ erg cm^3), if $a=b$ and $c=d$; and weak ($f_w \simeq 10^{-49}$ erg cm^3) otherwise. The strong interactions bind N's and or Λ's together to make π(NN), θ or τ (ΛN), etc. Then strong interactions are parity-conserving, while weak interactions are not, according to our theory.

RIASSUNTO (*)

Si discutono alcune conseguenze dell'ipotesi che nessuna interazione esistente in natura ci permette mai di determinare i segni delle masse di tutte le particelle elementari. L'ipotesi comporta la necessità di postulare l'invarianza di tutte le interazioni rispetto a un gruppo di trasformazioni che chiamiamo Inversione Individuale della Massa. L'interazione elettromagnetica e l'interazione π-N conservando la parità, sono compatibili con il postulato purchè quest'ultimo sia un accoppiamento PS(PV). La maggior parte delle interazioni di Fermi e le interazioni come ΛNπ e ΛNK non sono permesse se conservano la parità. Benchè alcune interazioni forti nella nostra teoria non conservino la parità, non sorge alcuna seria difficoltà se ridefiniamo uno stato di carica coniugata come suggerito da LANDAU, poichè la nostra teoria si dimostra invariante rispetto al prodotto della coniugazione della carica con la conversione dello spazio. La nostra teoria è equivalente per processi neutrinici alla teoria del neutrino a due componenti di T. D. LEE e C. N. YANG, eccetto che per il decadimento μ-e.

(*) *Traduzione a cura della Redazione.*

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Observations on the $\pi^+ \to \mu^+ \to e^+$ Angular Correlation in Nuclear Emulsion (*).

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(ricevuto l'8 Maggio 1957)

Summary. — The asymmetry coefficient characterizing the angular correlation in the π-μ-e decay has been measured in emulsion as -0.19 ± 0.06 for a sample of π^+-mesons produced at the Brookhaven Cosmotron. A discussion is given concerning depolarization effects in nuclear emulsion.

1. – Introduction.

Recently Lee and Yang (1) have predicted that if parity is not conserved in weak interactions, asymmetries in the angular distribution of certain decay processes may appear. In particular, a forward-backward asymmetry in the angular distribution of certain β-emitters should appear when they are polarized and the angular distribution of the electron in μ-β decay should show an asymmetry with respect to the momentum vector of the μ-meson emitted from a π-μ decay at rest. These predictions have been strikingly verified for the former case by Madame Wu and collaborators (2) and for the latter case by Lederman and coworkers (3) and by Telegdi and Friedman (4). We consider here some additional observation on the $\pi^+ \to \mu^+ \to e^+$ angular correlation as observed in nuclear emulsion.

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(+) American Association of University Women Post-Doctoral Research Fellow.

(1) T. D. Lee and C. N. Yang: *Phys. Rev.*, **104**, 256 (1956).

(2) Wu *et al.*: *Phys. Rev.*, **105**, 1413 (1957).

(3) R. L. Garwin, L. M. Lederman and M. Weinrich: *Phys. Rev.*, **105**, 1415 (1957).

(4) J. I. Friedman and V. L. Telegdi: *Phys. Rev.*, **105**, 1681 (1957).

2. – Theory.

According to the two-component neutrino theory of LEE and YANG [5], which is a particularly attractive parity non-conserving theory, the μ-meson resulting from the decay of a π-meson at rest will be completely longitudinally polarized. The spin of the μ-meson will be parallel (antiparallel) to its momentum if the ν emitted along with a μ-meson in the π decay is a neutrino (antineutrino) in the Lee-Yang convention. Further, in the subsequent μ-e decay the emitted electron is predicted to have an angular distribution relative to the μ-meson's spin of the form

$$\mathrm{d}N = p(\theta)\sin\theta\,\mathrm{d}\theta = (1 \pm a(\varepsilon)\cos\theta)\sin\theta\,\mathrm{d}\theta \ ,$$

in which the asymmetry a depends on the electron's energy. Hence the electron will have an asymmetry relative to the momentum of the μ-meson at emission given by $p(\theta) = 1 \pm a(\varepsilon)\cos\theta$. If one experimentally observes all electrons of energy greater than some minimum value ε_0, the angular distribution, integrated over the energy acceptance, will be

$$p(\theta) = 1 \pm \xi\alpha(\varepsilon_0)\cos\theta \qquad -1 \leqslant \xi \leqslant 1, \tag{1}$$

where ξ is a measure of the mixing of the vector and axial-vector couplings in the β-decay interaction; and $\alpha(\varepsilon_0)$ is plotted in Fig. 1. This energy dependence of the asymmetry predicted by the two component theory has been roughly confirmed by the preliminary evidence of LEDERMAN and coworkers at Columbia [3]. We shall use below Eq. (1) with the minus sign, so that ξ is a positive number.

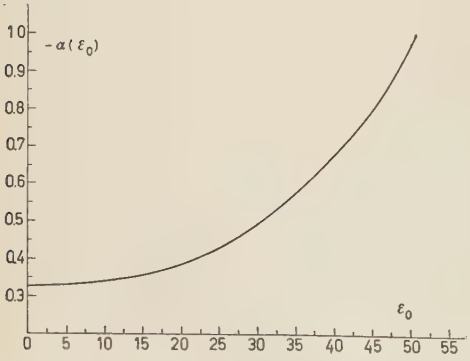

Fig. 1.

Experimentally the μ-meson is slowed to rest before it decays and one must therefore expect that the meson will be depolarized to some extent, i.e. that the interactions between the absorber and the μ-meson will rotate its spin, so that the spin at the time of decay is no longer in its original direction. However, since electric fields do not rotate the spin of a non-relativistic particle,

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[5] T. D. LEE and C. N. YANG: *Phys. Rev.*, **105**, 1671 (1957).

the nuclear Coulomb fields encountered by the μ-meson while slowing down have a very small effect on its spin, even though its momentum is appreciably deviated. Therefore the μ-meson comes to rest with its spin still in the direction of its initial momentum; for this reason the angular distribution of the decay electron is measured relative to the initial line of flight of the μ-meson, its final momentum being irrelevant. Subsequent processes in matter may yet depolarize the μ-meson, so that the observed asymmetry of the decay electrons will be reduced; the observed distribution will then be of the form $1+\overline{a}(\varepsilon_0)\cos\theta$, where $|\overline{a}(\varepsilon_0)| \leqslant |\xi\alpha(\varepsilon_0)|$. With the assumption of this form of angular distribution, $\overline{a}(\varepsilon_0)$ is determined from experiment by the usual forward-backward ratio $\overline{a}(\varepsilon_0) = 2(N_F - N_B)/(N_F + N_B)$ which has the standard deviation $\Delta\overline{a} = 2(1 + \overline{a}^2/8)/(N_F+N_B)^{\frac{1}{2}}$.

The experimentally determined parameter $\overline{a}(\varepsilon_0)$ can be related to the theoretical parameter $\xi\alpha(\varepsilon_0)$ if one knows the depolarization effects characteristic of the medium and if in addition the polarization of the incident μ-meson is known. If we define f to be the polarization of the incident μ beam (i.e. $1 - f$ is that fraction of the beam whose spin vectors are not correlated with $\boldsymbol{p}_\mu$) and if we denote by $\delta \leqslant 1$ the environmental effects leading to a diminution of the asymmetry (depolarization) we have $\overline{a}(\varepsilon_0) = f\delta\xi\alpha(\varepsilon_0)$. For example in emulsions where $\boldsymbol{p}_\mu$ is measured from a π decay at rest $f = 1$ according to the two-component theory.

3. – Experimental results.

To look for an asymmetry in the π^+-μ^+-e^+ chain in emulsion we have utilized a small stack of 2 in. $\times$ 3 in. $\times$ 400 μm Ilford G-5 stripped emulsions exposed at right angles to a carbon target in the external 2.9 GeV proton beam of the Brookhaven Cosmotron. Though at the time of exposure no measurements were made regarding the magnitude of stray magnetic fields, we believe that in addition to the Earth's field, any other fields present were certainly less than 0.5 G ([6]). The emulsions were scanned for π-μ decays of stopped π-mesons with the criterion that the μ stops in the same emulsion in which it was emitted but without requiring the observation of the μ-e decay. Of this class of π-μ decays, only those were accepted for analysis in which the μ came to rest at least 20 μm from either the air or glass surface of the emulsion. Of a total of 1070 π-μ decays satisfying the above criteria 1048 had the complete decay visible and measurable ([7]). For these events, the spatial

([6]) R. Adair: BNL Private communication.

([7]) Ten of the 22 cases were visible but had non-measurable secondaries due to an inability to sufficiently define the electron direction at emission in order to make a measurement; these correspond to quite low energy electrons.

angle between the μ-meson momentum at emission and the electron momentum at emission was measured. We found $N_F = 474$ and $N_B = 574$ yielding $\bar{a} = -0.19 \pm 0.06$. The angular distribution formed by collecting the data into eight equal intervals of $\cos\theta$ is shown in Fig. 2; the solid line is $(\frac{1}{2})(1 - 0.19\cos\theta)$. The agreement with a $(1+\bar{a}\cos\theta)$ distribution is quite striking.

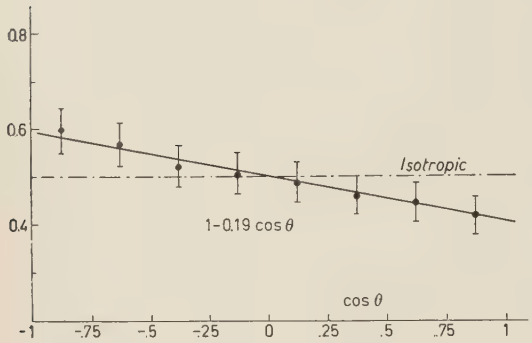

Fig. 2.

It is worthwhile to consider at this point whether the observational procedures employed could have introduced any bias into our results. The requirement that the complete decay occur in a single emulsion is principally one of convenience in measurement and of minimization of measurement errors (principally due to shrinkage). This requirement does tend to select a class of μ-mesons whose momentum vectors at emission lie in or near the plane of the emulsion, but this in itself will introduce no bias. However, the electron, leaving a minimum ionizing track, is harder to detect when steeply dipping rather than when it is flat in the plane of the emulsion. Thus, there will be a possible bias towards observing electron tracks parallel or antiparallel to the μ-meson tracks. If the loss occurs in the region $(\pi/2) - \alpha \leqslant \theta' \leqslant (\pi/2) + \alpha$, where θ' is the angle of the electron with respect to $\boldsymbol{p}_\mu$, then on the assumption that $\boldsymbol{p}_\mu$ lies principally in the emulsion plane, the corresponding value of $\bar{a}$ determined from the forward-backward ratio (which we denote by $\bar{a}'$) is

$$\bar{a}' = 2\,\frac{N_F - N_B}{N_F + N_B}\,\frac{1 - \eta\sin\alpha}{1 + \eta\sin^2\alpha},$$

where η is the fraction lost in the angular interval defined above. Thus for $\eta = 0.02$ and α as high as 45° the multiplicative factor is 0.995 corresponding to a correction of only 0.5%. Another bias which would discriminate between forward and backward electron tracks would arise from events in which the μ-meson decayed when on the point of leaving the emulsion, but this possibility is removed by the criterion that the μ-meson must stop at least 20 μm from either emulsion surface. Since in fact the electron was not visible in less than 2% of the events ([7]), such biases are a-posteriori seen to be small. There remains the question of the effective value of ε_0 (lowest electron energy accepted) for our observations in emulsion. We estimate that we accept with essentially 100% efficiency all electrons of energy greater than 5 MeV and this

conclusion is supported by consideration of the β-spectrum from μ-decay and the known number of observed but unmeasurable decays (7). We conclude that our experimental value of $\overline{a} = -0.19 \pm 0.06$ is not influenced by observational bias and corresponds practically to a zero cut-off energy.

4. – Discussion.

The parameter of most immediate physical interest in the two-component neutrino theory is the mixture parameter ξ. This is related to the measured coefficient $\overline{a}(\varepsilon_0)$ via $\xi = (\overline{a}(\varepsilon_0))/(f\delta\alpha(\varepsilon_0))$; thus since f, $\delta \leqslant 1$, we obtain a lower limit $\xi \geqslant \overline{a}(\varepsilon_0)/\alpha(\varepsilon_0)$. From our experiment we find $\xi \geqslant 0.39$; on the other hand the Columbia experiments (8) on carbon extrapolated to zero cut-off energy yield $\overline{a}(0) = -0.29 \pm 0.02$ or $\xi \geqslant 0.81$, while their experiments with emulsion, extrapolated to zero cut-off energy yield $\overline{a}(0) = -0.14 \pm 0.03$. It is of some interest to compare the two results with emulsion obtained in these different ways, for in principle this would allow a determination of the polarization of the μ-meson beam used by the Columbia group. This follows from the observation that for the emulsion experiments in which the π-μ-e chain is observed, $f = 1$ and thus $\overline{a}_c/\overline{a}_e = (f_c\delta_c\alpha_c(0)\xi)/(\delta_e\alpha_e(\varepsilon_0)\xi)$, giving $f_c = (\delta_e\overline{a}_c)/(\delta_c\overline{a}_e)$, where the subscript c means the Columbia type emulsion experiment extrapolated to 0 cut-off energy (the difference between a 0 and a 5 MeV cut-off energy in the value of α is negligible) and e refers to the type reported here. If the assumptions are made: (i) that the depolarization effects which occur do not set in until the μ-energy is less than several MeV; (ii) that these effects occur in times short compared to 1 μs (this is necessary since the Columbia counter experiment utilized a time gate extending from 0.75 to 2 μs) and (iii) that the depolarization effects in emulsion are not sensitive to variations in constitution which may occur from batch to batch, then $\delta_e = \delta_c$ and $f_c = \overline{a}_c/\overline{a}_e$. If for emulsion we use a weighted mean of the Chicago results (9) ($\overline{a} = -0.174 \pm 0.038$), the Göttingen (10) results obtained with emulsions exposed at Chicago ($\overline{a} = -0.095 \pm 0.044$) and those reported here, we find $\overline{a}_e = -0.15 \pm 0.025$; this yields $f_c = 0.93 \pm 0.25$. This result applied to the carbon experiment (with $\delta = 1$) gives the inequality $0.81 \leqslant \xi \leqslant 1$.

The meaningfulness of the result reached above thus depends on the validity of assumptions (i), (ii) and (iii) and we now consider them. Assumption (i) is easily justified by the calculation of the depolarization in flight. Wen-

(8) L. Lederman: Private communication.
(9) J. I. Friedman: Private communication.
(10) N. Biswas *et al.*: Preprint.

TZEL ([11]) gives a simple derivation to lowest order in v/c of the depolarization by Coulomb scattering. One concludes that for a μ-meson slowed to rest from an initial velocity v, the mean square angle of rotation of the spin is $\langle\theta^2\rangle = (Z/32)(m_e/m_\mu)(v/c)^4$. For μ-mesons emitted from π-mesons at rest, this rotation is utterly negligible; for the Columbia μ-mesons $v \sim 0.5\ c$ on exit from the telescope incident on the target which gives $\langle\theta^2\rangle \sim 0.6 \cdot 10^{-4}$. Thus in either case the depolarization of the meson in flight is negligible and assumption (i) is justified.

To facilitate the discussion of assumption (ii) let us first summarize possible depolarization processes. Depolarization at rest will occur through the action of magnetic fields, which rotate the spin with the Larmor frequency, $\omega \approx 10^5\, H\ \mathrm{s}^{-1}$ (H in gauss) for the μ-meson, taking it to have the normal Dirac moment as shown by LEDERMAN ([3]). Thus a magnetic field will depolarize a μ-meson in a time $\omega^{-1} = 10^{-5}$ s/G. Since the average value, at the time of decay, of a component of spin perpendicular to a field is reduced by a factor $(1+\omega^2\tau^2)^{-1}$ from its initial value, a critical field is H_τ such that $\omega^{-1} = \tau_\mu = 2 \cdot 10^{-6}$ s; one finds $H_\tau = 5$ G. If the magnetic field is not constant, then its depolarization effect will be smaller. If the field acting on the μ-meson is nearly constant for times of order T (« relaxation time »), then we have $(\mathrm{d}/\mathrm{d}t)(\langle\theta^2\rangle) \approx \langle\omega^2\rangle T$ where $\langle\omega^2\rangle$ is the mean square vector Larmor frequency. Thus in order that even a completely paramagnetic substance ($H \sim 10^4$ G) depolarize a μ-meson, T must be greater than approximately $0.5 \cdot 10^{-12}$ s. This figure is about ten times the collision frequency of the μ-meson at room temperature.

A very strong magnetic field acts on a positive μ-meson if it captures an electron to form a « muonium » atom ([12,13]). The magnetic spin-spin interaction between the electron and the meson splits the singlet and triplet spin states by an energy $\hbar\,\Delta\omega$; in the $1S$ ground state $\Delta\omega = 3.8 \cdot 10^{10}\ \mathrm{s}^{-1}$. If a μ-meson captures an electron with antiparallel spin, the state formed is a linear combination of singlet and triplet states. Since the relative phase of these states oscillates with a frequency $\Delta\omega$, the spin of the μ-meson is flipped with this same frequency. Thus the formation of muonium in the $m_J = 0$ state depolarizes the μ-meson in a time very short compared to a microsecond. However, the μ-meson has an equal chance to capture an electron with

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([11]) G. WENTZEL: *Phys. Rev.*, **75**, 1810 (1949).

([12]) This was suggested to WENTZEL by RABI (see footnote ([10])). The name « muonium » should perhaps have been reserved for $(\mu^+\mu^-)$; however it seems already to have been borrowed.

([13]) The stability of muonium in an ionic crystal is inferred from the experiment of DELBECQ, SMALLER and YUSTER [*Phys. Rev.*, **104**, 599 (1956)] on KCl-KH crystals. When suitably treated, this additively colored crystal shows paramagnetic resonance characteristic of atomic hydrogen.

parallel spin to form (pure) triplet muonium; this does not in itself depolarize the μ-meson. However, for external fields varying more slowly than $\Delta\omega$ the spins of the electron and meson are now coupled, so that the large electronic moment causes the triplet muonium atom to precess 100 times faster than a bare μ-meson in the same field. Thus, the critical field, the value of a constant field which will depolarize a μ-meson bound in triplet muonium, is only 0.04 G.

The above observations tend to throw doubt on muonium formation as the mechanism for depolarization, for although the Chicago emulsion experiment was done in an external field of less than 10^{-2} G, their observed asymmetry is no larger than that observed by Columbia or by us, where there was at least the Earth's field present. A way out is to assume that triplet muonium is depolarized or converted to singlet by local fields, so that the presence of an external field is irrelevant. For instance, there are the nuclear magnetic fields in AgBr, which are of the order of a gauss, although a rough estimate of the relaxation time ([14]) finds it to be a factor of ten too short for these fields to cause depolarization or conversion. However, even if this estimate is wrong, or there is some other mechanism which depolarizes or converts triplet muonium, a problem remains. Why is the depolarization not complete in emulsion? This problem arises partly from the expectation that a meson slowing to rest in matter will nearly always have captured an electron as it comes to rest. More directly, we have the observations of Lederman and coworkers that depolarization is nearly complete in a bulk ionic crystal, NaCl. ([8]). This suggests that the depolarization is only 50% complete in emulsion simply because the μ-meson comes to rest in the AgBr grains less than 50% of the time. This behavior is of course different from that known for the negative μ-meson, which is found to be captured by Ag or Br roughly 70% of the time, following the Fermi-Teller law ([15]), which says that the probability of stopping in a phase of material is proportional to the number of electrons in that phase. This law is reasonable for the negative meson, even for velocities so small that ordinary ionization energy loss does not occur, for there is still the « adiabatic » ionization process ([16]) in which the mere presence of the negatively charged meson inside an atom produces ionization. Such a process is, however, not available to a slow positive meson, which can only lose energy to the recoil of atoms. Since the atoms of the gelatine phase are lighter than Ag or Br, these are more effective in slowing the positive meson to rest.

It is difficult to find any mechanism except muonium formation to explain appreciable depolarizations. Although the formation of singlet muonium provides a « quick » depolarization of the μ-meson, the equal amount of depola-

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([14]) This is made from the observation of Delbecq *et al.* of the temperature at which interstitial hydrogen becomes mobile in KCl.

([15]) E. Fermi and E. Teller: *Phys. Rev.*, **72**, 399 (1947).

([16]) A. S. Wightman: *Phys. Rev.*, **77**, 521 (1950).

rization through the formation of triplet muonium (which it was necessary to assume in order to understand the agreement of Chicago's results with Columbia's and ours) is a « slow » process which violates assumption (ii). Further it is possible that assumption (iii) is violated if indeed the μ^+-mesons tend not to stop in the AgBr grains. This is because the meson will most probably be in a AgBr grain when it is slowed by ionization to a velocity at which it no longer ionizes; its remaining path length in AgBr turns out to be just the order of a grain. Thus the probability of the μ-meson getting out of the grain before it comes to rest could be strongly grain size dependent. In lieu of any information on this particular aspect of the depolarization phenomenon, we are unable to reach any firm conclusion at the present, and must let the question be subject to further experiment. In this context, we note in particular the rather divergent results of the Chicago and Göttingen groups using a similar exposure setup, as well as the widely varying results on the π-μ-e asymmetry as observed in emulsions exposed to the cosmic radiation reported by the Minnesota group ([17]) ($\bar{a} = -0.03 \pm 0.04$), the Bristol ([18]) group ($\bar{a} = -0.08 \pm 0.05$, this is an average over several different cosmic ray exposures) and the Rome ([19]) group ($\bar{a} = -0.222 \pm 0.067$). These divergences may be viewed as contributing supporting evidence to the view that the determination of the asymmetry parameter in emulsion is principally a reflection of variation in the fine details of the emulsion constitution, or one may take the viewpoint as suggested by the Minnesota group, that these represent information relevant to the properties of the π-mesons accepted in the sample. Only further experiments can answer the very interesting questions posed by the data available at present.

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We should like to express our appreciation to Drs. G. B. Collins and E. O. Salant and the staff of the Cosmotron Laboratory for their assistance in obtaining the emulsion exposure and to our scanning staff for their aid in obtaining the π-μ decays used in the analysis.

([17]) P. Fowler *et al.*: Preprint.
([18]) B. Bhowmik *et al.*: Preprint.
([19]) C. Castagnoli *et al.*: Preprint.

RIASSUNTO (*)

Il coefficiente di asimmetria che caratterizza la correlazione angolare nel decadimento π-μ-e è stato misurato in emulsione su un campione di mesoni π^+ prodotti dal cosmotrone di Brookhaven risultando -0.19 ± 0.06. Segue una discussione sugli effetti di depolarizzazione nell'emulsione nucleare.

(*) *Traduzione a cura della Redazione.*

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On the Formulation of Quantized Field Theories - II.

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Summary. — We discuss the concept of causal scattering matrices using retarded multiple commutators of field operators.

1. – Scattering matrix and field operators.

One of the main problems of relativistic quantum theory is to obtain information about scattering matrix elements. To this end it is customary to start from a quantized field theory; i.e. operator fields which obey certain equations and commutation relations. On the other hand one can try — following Heisenberg's original suggestion — to base a theory of interacting particles entirely on requirements imposed on the scattering matrix.

In what follows we intend to discuss the connection between such an S-matrix formalism and conventional quantized field theories.

Our aim is

a) to clarify some concepts employed in recent work; in particular to define rather precisely what is usually considered a « causal » scattering matrix;

b) to investigate some expressions which seem to be useful if one wants to derive causality conditions for a scattering matrix.

We begin with a pure S-matrix formalism [1]. No interacting field operators are considered but only an operator S which describes the scattering,

[1] W. Heisenberg: *Zeits. f. Phys.*, **120**, 513, 673 (1943).

production and annihilation of particles. Masses, spins etc. of all stable particles contained in the model are given parameters (2). First we require only that S describes a physical system in accordance with the general principles of quantum theory — with the exception of causality — and of special relativity. These concepts and their consequences for our problem have been discussed by HAAG (3) and this chapter is based on his work. The above principles imply: There exists a unitary representation of the inhomogenous Lorentz group in the Hilbert space of the state vectors.

The results of observations in the distant past or future may be characterized by states corresponding to non-interacting (incoming or outgoing) particles. The operator S gives a unitary mapping between initial and final states and therefore transition probabilities may be computed from S in the usual manner. For simplicity of notation we discuss a model containing only one kind of stable particle having mass m, spin 0 and no charge. For this case we give first the general form of S which follows from the mentioned principles. It is convenient to introduce the phase matrix η to take the unitarity of S into account.

$$S = \exp[i\eta] . \tag{1}$$

Then η is a hermitian operator which has the following structure:

$$\eta = \sum_{n=4}^{\infty} \frac{1}{n!} \int \mathrm{d}x_1 \ldots \mathrm{d}x_n h_n(x_1, \ldots, x_n) : A_{\mathrm{in}}(x_1) \ldots A_{\mathrm{in}}(x_n) : . \tag{2}$$

The operator field $A_{\mathrm{in}}(x)$ describes the incoming particles:

$$(\Box - m^2) A_{\mathrm{in}}(x) = 0 , \qquad [A_{\mathrm{in}}(x), A_{\mathrm{in}}(y)] = i\Delta(x - y) . \tag{3}$$

The double dots in (2) stand as usual for the Wick product. The functions $h_n(x_1, \ldots, x_n)$ are c-numbers. They have to satisfy some general requirements. To discuss these it is useful to transform (2) into momentum space.

Let

$$A_{\mathrm{in}}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}^4k \exp[ikx]\, \delta(k^2 + m^2) A_{\mathrm{in}}(k) , \tag{4}$$

$$h_n(x_1, \ldots, x_n) = \tag{5}$$

$$= \frac{1}{(2\pi)^{\frac{3}{2}n}} \int \mathrm{d}k_1 \ldots \mathrm{d}k_n \exp[-ik_1x_1 - \ldots - ik_nx_n]\, \delta(k_1 + \ldots + k_n) \tilde{h}_n(k_1, \ldots, k_n) \quad (4) .$$

(2) We do not attempt to distinguish between « elementary » and « bound » particles.

(3) R. HAAG: *Dan. Mat. Fys. Medd.*, **29**, no. 12 (1955).

(4) The factor $\delta(k_1 + \ldots + k_n)$ is of course a consequence of translation invariance and expresses energy-momentum conservation.

Then (2) becomes:

$$(2') \qquad \eta = \sum_{n=4}^{\infty} \frac{1}{n!} \int \mathrm{d}k_1 \ldots \mathrm{d}k_n \, \delta(k_1 + \ldots + k_n) \cdot$$

$$\cdot \tilde{h}_n(k_1, \ldots, k_n) \, \delta(k_1^2 + m^2) \ldots \delta(k_n^2 + m^2) : A_{\mathrm{in}}(k_1) \ldots A_{\mathrm{in}}(k_n) : .$$

Thus only the values of $\tilde{h}_n(k_1, \ldots, k_n)$ on the « energy shell » (mass hyperboloid) appear in this formula for η. Outside the energy shell the functions can be chosen in an arbitrary manner; i.e. $h_n(x_1, \ldots, x_n)$ is by no means uniquely determined if η is given. On the energy shell the function $\tilde{h}_n(k_1, \ldots, k_n)$ must have the following properties:

a) $\tilde{h}_n(k_1, \ldots, k_n)$ is a symmetric function of $k_1, \ldots, k_n$;

b) $\tilde{h}_n^*(k_1, \ldots, k_n) = \tilde{h}_n(-k_1, \ldots, -k_n)$ (this is equivalent to the hermiticity of η);

c) $\tilde{h}_n(k_1, \ldots, k_n)$ is invariant with respect to the homogenous Lorentz group;

d) $\tilde{h}_n(k_1, \ldots, k_n)$ contains no four-dimensional δ-functions. (Otherwise, not all observable quantities like cross-sections and their generalization for many-particle processes would be finite);

e) $\tilde{h}_n(k_1, \ldots, k_n)$ is a continuous function of its invariant variables. (Continuity of scattering matrix elements);

f) $\tilde{h}_n(k_1, \ldots, k_n) = \tilde{h}_n(-k_1, \ldots, -k_n)$ (i.e. $\tilde{h}_n(k_1, \ldots, k_n)$ real, because of (*b*)) follows if we require invariance against space-time inversion. If the theory is invariant under space or time inversion alone, then $\tilde{h}_n$ is invariant under $\boldsymbol{k}_i \to -\boldsymbol{k}_i$ or $k_{i0} \to -k_{i0}$.

All these are requirements on the energy shell. Since the extrapolation off the shell is so far arbitrary we can demand — and this will be done in what follows — that the functions satisfy the conditions *a)-f)* everywhere. Then we can also formulate these requirements in terms of $h_n(x_1, \ldots, x_n)$ by Fourier transformation. The listed properties — which leave an enormous degree of arbitrariness — contain everything (as far as we know) that general physical principles, with the exception of causality, require in such a model.

We consider now the connection between such a scattering matrix and Lorentz invariant operator fields. In the framework of S-matrix formalism we have the relation

$$(6) \qquad A_{\mathrm{out}}(x) = S^{\dagger} A_{\mathrm{in}}(x) S \, .$$

We shall show: For any given S-matrix there exist (many!) invariant fields $A(x)$ which tend asymptotically for $t \to \pm \infty$ to the given fields $A_{\mathrm{out}}(x)$ resp. $A_{\mathrm{in}}(x)$.

A precise formulation of this asymptotic behavior is [5]:

$$\lim_{t\to\pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A^f_{\text{out}}\Psi) \,. \tag{7}$$

In this formula

$$A^f(t) = +\, i \int \mathrm{d}^3x \left\{ A(x) \frac{\partial}{\partial x_0} f^*(x) - \frac{\partial}{\partial x_0} A(x) f^*(x) \right\} \text{ [6]} \,,$$

(the operators A^f_{out}, A^f_{in} defined in the same manner are time independent) where $f(x)$ is an arbitrary normalizable solution of the Klein-Gordon equation.

We have, according to (6):

$$\begin{aligned} A_{\text{out}}(x) &= \exp[-i\eta] A_{\text{in}}(x) \exp[i\eta] \\ &= A_{\text{in}}(x) + i \int_0^1 \mathrm{d}\lambda \exp[-i\lambda\eta] [A_{\text{in}}(x), \eta] \exp[i\lambda\eta] \\ &= A_{\text{in}}(x) - \int \mathrm{d}^4y \, \Delta(x-y) \int_0^1 \mathrm{d}\lambda \exp[-i\lambda\eta] \vartheta(y) \exp[i\lambda\eta] \,. \end{aligned} \tag{8}$$

The relation

$$[A_{\text{in}}(x), \eta] = i \int \Delta(x-y) \vartheta(y) \, \mathrm{d}^4y \,, \tag{9}$$

with

$$\vartheta(y) = \sum_{n=2}^{\infty} \frac{1}{n!} \int \mathrm{d}x_1 \ldots \mathrm{d}x_n h_{n+1}(y, x_1, \ldots, x_n) : A_{\text{in}}(x_1) \ldots A_{\text{in}}(x_n) : \,, \tag{10}$$

has been used [7,8].

Now we introduce interacting field operators:

$$A(x) = A_{\text{in}}(x) + \int \mathrm{d}^4y \, \Delta_R(x-y) \int_0^1 \mathrm{d}\lambda \exp[-i\lambda\eta] \vartheta(y) \exp[i\lambda\eta] \,. \tag{11}$$

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[5] Cf. H. LEHMANN, K. SYMANZIK, W. ZIMMERMANN: *Nuovo Cimento*, **1**, 425 (1955); in the following quoted as LSZ I.

[6] GREENBERG and WIGHTMAN (unpublished manuscript) have pointed out that the operator $A^f(t)$ may lead to states with infinite norm. To avoid this, they introduce time-averaged operators. This modification does not change the results derived in this paper. We wish to thank Drs. GREENBERG and WIGHTMAN for informing us of their results prior to publication.

[7] $\vartheta(y) = \delta\eta/\delta A_{\text{in}}(y)$ is the functional derivative of η with respect to $A_{\text{in}}(y)$, however, we shall not need this characterisation.

[8] The function $h_3(y; x_1, x_2)$ included in the sum is of course zero on the energy shell. It appears only in the extrapolation process.

It should be noted here that only

$$\int \Delta(x-y)\vartheta(y)\,\mathrm{d}^4y \qquad \text{but not} \qquad \int \Delta_R(x-y)\vartheta(y)\,\mathrm{d}^4y\,,$$

is uniquely determined if η is given. For it is clear from (10) that $\tilde{h}_{n+1}(k, k_1, \ldots, k_n)$ has to be defined for $k^2 + m^2 \neq 0$ to determine $\vartheta(\eta)$. As has been discussed this extrapolation outside the energy shell can be carried out in different ways; we demand only that the conditions which $\tilde{h}_{n+1}(k, k_1, \ldots, k_n)$ satisfies on the energy shell be fulfilled everywhere. Then it follows that $A(x)$ is invariant and hermitian. Moreover it satisfies the asymptotic condition (7) as is easily shown by using the Riemann-Lebesgue lemma on Fourier integrals.

We wish to stress that the described procedure of constructing relativistic field operators $A(x)$ corresponding to a given S-matrix does not involve any divergent steps. The necessary extrapolation of the functions $\tilde{h}_n(k_1, \ldots, k_n)$ can of course be accomplished in a continuous and therefore finite manner.

Using the definition

$$j(x) = -\int_0^1 \mathrm{d}\lambda \exp[-i\lambda\eta]\vartheta(x)\exp[i\lambda\eta]\,, \tag{12}$$

we can write:

$$\begin{cases} A_{\mathrm{out}}(x) = A_{\mathrm{in}}(x) + \int \Delta(x-y)j(y)\,\mathrm{d}y\,, \\ A(x) \quad = A_{\mathrm{in}}(x) - \int \Delta_R(x-y)j(y)\,\mathrm{d}y\,, \\ (\Box - m^2)A(x) = j(x)\,. \end{cases} \tag{13}$$

Using the developed formalism we turn now to the concept of a causal scattering matrix. As is customary we call a field operator $A(x)$ causal — without discussing here the physical interpretation — if

$$[A(x), A(y)] = 0\,. \qquad \text{for } (x-y)^2 > 0. \tag{14}$$

The operators $A(x)$ defined by (11) will of course not generally satisfy this condition. It is essential now that (14) is not only a condition on the extrapolation off the energy shell but also a condition for the scattering matrix, i.e. for the functions $\tilde{h}_n(k_1, \ldots, k_n)$ for $k_1^2 = \ldots = k_n^2 = -m^2$. If these functions have only the properties listed above, then it is in general impossible to extrapolate them continuously in such a manner that (14) is satisfied. This represents an additional requirement.

Hence we define a scattering matrix as causal if there exists at least one (continuous) extrapolation off the energy shell such that the resulting field $A(x)$ satisfies the commutator condition. It is an entirely open question whether any scattering matrix exists which is causal in the sense of this definition or whether this is too stringent a demand. Our notion of a causal S-matrix is equivalent to what is being used in investigations of local field theories. However, the above definition shows clearly that it must be possible in principle to express all consequences of the causality condition (14) for the energy shell in terms of S-matrix elements only.

We conclude this section by giving some formulae needed for the following. They are taken from LSZ I using a slightly different notation.

Let $\{f_\alpha(x)\}$ denote a complete and orthonormal system of positive frequency solutions to the Klein-Gordon equation.

$$(15) \quad -i\int \mathrm{d}^3x f_\alpha(x)\overleftrightarrow{\frac{\partial}{\partial x_0}} f^*_\beta(x) = -i\int \mathrm{d}^3x \left\{f_\alpha(x)\frac{\partial}{\partial x_0} f^*_\beta(x) - \frac{\partial}{\partial x_0} f_\alpha(x)\cdot f^*_\beta(x)\right\} = \delta_{\alpha\beta}\,,$$

$$(16) \quad \sum_{\alpha=1}^{\infty} f_\alpha(x) f^*_\alpha(x') = i\Delta^+(x-x')\,.$$

The field operator $A(x)$ may be developed with respect to these functions

$$(17) \quad A(x) = \sum_{\alpha=1}^{\infty}\left\{f_\alpha(x)A^\alpha(t) + f^*_\alpha(x)A^{\alpha*}(t)\right\}.$$

The coefficients are given by

$$(18) \quad A^\alpha(t) = i\int \mathrm{d}^3x A(x)\overleftrightarrow{\frac{\partial}{\partial x_0}} f^*_\alpha(x)\,.$$

The corresponding operators $A^\alpha_{\mathrm{in}}(t)$ and $A^\alpha_{\mathrm{out}}(t)$ are of course time-independent:

$$A^\alpha_{\mathrm{in}}(t) = A^\alpha_{\mathrm{in}}\,, \qquad A^\alpha_{\mathrm{out}}(t) = A^\alpha_{\mathrm{out}}\,.$$

They constitute the limiting values of $A_\alpha(t)$ for $t \to \pm\infty$

$$(19) \quad \lim_{t\to\pm\infty} (\Phi, A_\alpha(t)\Psi) = (\Phi, A^\alpha_{\substack{\mathrm{out}\\ \mathrm{in}}}\Psi)\,.$$

The operators A^α_{in}, A^α_{out} satisfy the commutation relations:

$$(20) \quad \begin{cases} [A^\alpha_{\mathrm{in}}, A^{\beta*}_{\mathrm{in}}] = [A^\alpha_{\mathrm{out}}, A^{\beta*}_{\mathrm{out}}] = \delta_{\alpha\beta}\,, \\ [A^\alpha_{\mathrm{in}}, A^\beta_{\mathrm{in}}] = [A^\alpha_{\mathrm{out}}, A^\beta_{\mathrm{out}}] = 0\,. \end{cases}$$

$A_{\text{in}}^{\alpha *}$ creates an incoming particle with wave function $f_\alpha(x)$ and may be used to construct the complete orthonormal system

$$\Phi_{\text{in}}^{\alpha_1 \dots \alpha_k} = \frac{1}{\sqrt{p_{\alpha_1 \dots \alpha_k}}} A_{\text{in}}^{\alpha_1 *} \dots A_{\text{in}}^{\alpha_n *} \Omega \,, \tag{21}$$

$(p_{\alpha_1 \dots \alpha_k} = n_1! \dots n_r!$ if n_j of the indices α_i are equal).

For an exact formulation of the asymptotic behavior it is essential to work with normalizable solutions of the Klein-Gordon equations (wave packets). For this reason we use a discrete orthonormal system in direct applications of the asymptotic condition. Final results can be expressed by means of plane waves without introducing errors. The development of $A_{\text{in}}(x)$ with respect to plane waves is given by (4). We define $a_{\text{in}}(k)$ by

$$a_{\text{in}}(k) = A_{\text{in}}(k) \,, \qquad a_{\text{in}}^*(k) = A_{\text{in}}(-k) \,. \qquad \text{for } k_0 > 0 \,. \tag{22}$$

It satisfies the invariant relations

$$[a_{\text{in}}(k),\, a_{\text{in}}^*(k')] = 2k_0\, \delta(\boldsymbol{k} - \boldsymbol{k}') \,, \qquad [a_{\text{in}}(k),\, a_{\text{in}}(k')] = 0 \,, \tag{23}$$

$a_{\text{out}}(k)$ is defined in the same manner.

Finally, for the S-matrix we use the notation:

$$(\Phi_{\text{in}}^{p_1 \dots p_k},\, S\Phi_{\text{in}}^{q_1 \dots q_l}) = (\Phi_{\text{out}}^{p_1 \dots p_k},\, \Phi_{\text{in}}^{q_1 \dots q_l}) \,, \tag{24}$$

with

$$\Phi_{\substack{\text{out} \\ \text{in}}}^{p_1 \dots p_n} = a_{\substack{\text{out} \\ \text{in}}}^*(p_1) \dots a_{\substack{\text{out} \\ \text{in}}}^*(p_n) \Omega \,.$$

2. – Definition of retarded products.

We have defined an S-matrix to be causal if there exists an extrapolation such that the commutator condition (14) is satisfied. So far this is a formal definition that includes the S-matrices of the conventional local field theories.

However, we do not know if this concept of causality is the one realized in nature. Therefore it is necessary to confront the properties of (in the above sense) causal S-matrices with experiment. The commutator condition (14) does not immediately yield explicit properties of the scattering matrix; non-trivial mathematical steps must be taken to derive practically useful consequences from it. We shall discuss some formal concepts which are useful — and partly

have been used, especially by POLKINGHORNE [9] — for an evaluation of the commutator condition.

In this section we define and investigate a retarded product of field operators. For causality problems this product seems more promising than e. g. the time-ordered product. We define the retarded product of *two* field operators as the retarded commutator:

$$R(x;\, y) = -\, i\theta(x-y)[A(x),\, A(y)]\,. \tag{26}$$

This, expression is familiar from the theory of dispersion relations. The retarded product of $n+1$ field operators is defined by

$$\begin{cases} n=0: & R(x) = A(x)\,, \\ n\geqslant 1: & R(x;\, x_1 \ldots x_n) = \\ & = (-\,i)^n \sum \theta(x-x_1) \ldots \theta(x_{n-1}-x_n)[\ldots[A(x),\, A(x_1)] \ldots A(x_n)], \end{cases} \tag{27}$$

which will turn out to be a natural generalization of (26). The summation is taken over all permutations of the n coordinates x_i.

We call the vacuumexpectation values

$$r(x;\, x_1 \ldots x_n) = (\Omega,\, R(x;\, x_1 \ldots x_n)\Omega)\,, \tag{28}$$

retarded functions (r-functions) of the field operators. Three simple properties of R-products follow from the definition:

Retardation: $R(x;\, x_1, \ldots x_n)$ vanishes if one or several of the times x_{i0} is larger than x_0.

Symmetry: $R(x;\, x_1, \ldots x_n)$ is symmetric in the co-ordinates x_i.

Hermiticity: The R-product of hermitian operators is hermitian; i.e. the retarded functions are real.

The R-products can be generated from the functional [10]

$$\mathcal{R}\{x;\, J\} = -\, i\mathcal{C}^*\{J\}\, \frac{\delta}{\delta J(x)}\, \mathcal{C}\{J\}; \qquad \mathcal{C}\{J\} = T \exp\left[\left(i \int \mathrm{d}x J(x) A(x)\right)\right]. \tag{29}$$

[9] POLKINGHORNE (*Nuovo Cimento*, **4**, 216 (1956) and *Proc. Cambr. Phil. Soc.* **53**, 260 (1957)) has introduced a general type of product which depends not only on the space-time coordinates of the field operators, but also on additional parameters. For special choice of these parameters Polkinghorne's product is identical with our R-product, as defined below in eq. (27).

[10] Cf. LSZ I, appendix.

by means of functional differentiation with respect to $J(x)$.

$$R(x; x_1 \dots x_n) = \frac{\delta^n \mathcal{R}\{x, J\}}{\delta J(x_1) \dots \delta J(x_n)}\Big|_{J\ 0} . \tag{30}$$

We state some more properties of R-products which will then be used to discuss the significance of R-products for the derivation of causality conditions.

1) Invariance properties.

$$\begin{cases} R(x + a; x_1 + a, \dots, x_n + a) = \exp[-iPa] R(x; x_1 \dots x_n) \exp[iPa] , \\ \text{if} \quad A(x + a) = \exp[-iPa] A(x) \exp[iPa] . \end{cases} \tag{31}$$

The retarded functions depend therefore only on the coordinate differences:

$$r(x; x_1 \dots x_n) = r(x - x_1, \dots, x - x_n) . \tag{32}$$

The retarded product of Lorentz invariant and causal field operators is moreover invariant under proper Lorentz transformations L:

$$\begin{cases} R(Lx; Lx_1, \dots, Lx_n) = U(L) R(x; x_1 \dots x_n) U(L)^{-1} , \\ r(Lx; Lx_1, \dots, Lx_n) = r(x; x_1 \dots x_n) , \end{cases} \tag{33}$$

if $A(Lx) = U() A(x) U(L)^{-1}$ and $[A(x) A(y)] = 0$ for $(x - y)^2 > 0$. (33) is proved using (29) or applying Jacobi identities to (27).

2) The asymptotic condition at $t = -\infty$ gives rise to the following commutation relations of R-products with the incoming field operator: $(K_z = \Box_z - m^2)$

$$[R(x; x_1 \dots x_n), A_{\text{in}}(z)] = -i \int \mathrm{d}z' \Delta(z - z') K_{z'} R(x; x_1 \dots x_n, z'), \tag{34}$$

or

$$[R(x; x_1 \dots x_n), A^*_{\text{in}}(k)] = \frac{\varepsilon(k)}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}z \exp[ikz] K_z R(x; x_1 \dots x_n, z) . \tag{34'}$$

To give a rigorous proof of these relations we use a discrete orthonormal system $\{f_\alpha(x)\}$ (cfr. (17)). Then (3) takes the form

$$[R(x; x_1 \dots x_n), A^{\alpha *}_{\text{in}}] = \int \mathrm{d}z\, f_\alpha(x) K_z R(x; x_1 \dots x_n, z) \ (^{11,12}) .$$

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(11) K. NISHIJIMA (to be published) has suggested a formulation of the asymptotic condition which avoids the operators of incoming and outgoing fields and may be ob-

Now we note that as a consequence of the asymptotic condition at $t = -\infty$ we have for arbitrary normalizable states:

$$
\begin{aligned}
(\Phi, [R(x; x_1 \dots x_n), A^{\alpha *}_{\text{in}}]\Psi) &= \lim_{z_0 \to -\infty} (\Phi, [R(x; x_1 \dots x_n), A^{\alpha *}(z_0)]\Psi) = \\
&= \lim_{z_0 \to -\infty} i \int \mathrm{d}^3 z (\Phi, [R(x; x_1 \dots x_n) A(z)]\Psi) \overleftrightarrow{\frac{\partial}{\partial z_0}} f_\alpha(z) = \\
&= - \lim_{z_0 \to -\infty} \int \mathrm{d}^3 z (\Phi, R(x; x_1 \dots x_n, z)\Psi) \overleftrightarrow{\frac{\partial}{\partial z_0}} f_\alpha(z) = \\
&= \int \mathrm{d}z \frac{\partial}{\partial z_0} \left\{ (\Phi, R(x; x_1 \dots x_n, z)\Psi) \overleftrightarrow{\frac{\partial}{\partial z_0}} f_\alpha(z) \right\} = \\
&= \int \mathrm{d}z \, f_\alpha(z) K_z (\Phi, R(x; x_1 \dots x_n, z)\Psi),
\end{aligned}
$$

since the boundary term

$$
\lim_{z_0 \to +\infty} \int \mathrm{d}^3 z (\Phi, R(x; x_1 \dots x_n z)\Psi) \overleftrightarrow{\frac{\partial}{\partial z_0}} f_\alpha(z) ,
$$

vanishes due to the retarded character of the R-product.

Iteration of (34′) gives the result:

$$
\begin{aligned}
(35) \qquad & \varepsilon(k_1) \dots \varepsilon(k_n) [\dots [R(x; x_1 \dots x_m), A^*_{\text{in}}(k_1)] \dots A^*_{\text{in}}(k_n)] = \\
& = \frac{1}{(2\pi)^{\frac{3}{2}n}} \int \mathrm{d}z_1 \dots \mathrm{d}z_n \exp [i \sum k_i z_i] K_{z_1} \dots K_{z_n} R(x; x_1 \dots x_m z_1 \dots z_n).
\end{aligned}
$$

3) Using the asymptotic condition both at $t = -\infty$ and $t = +\infty$ we obtain commutation relations of the outgoing field with the incoming field [13]:

$$
(36) \qquad [A_{\text{out}}(x) A_{\text{in}}(y)] = i\Delta(x - y) - i \int \mathrm{d}x' \, \mathrm{d}y' \, \Delta(x - x') \Delta(y - y') K_{x'} K_{y'} R(x'; y') .
$$

tained by taking matrix elements of (34″) with respect to $\Phi^{\alpha_1 \dots \alpha_l}_{\text{in}}$, $\Phi^{\beta_1 \dots \beta_l}_{\text{in}}$. This modified procedure is useful to include the possibility of bound states in the formalism of r-functions, especially in connection with the completeness of the incoming and outgoing states (compare Appendix). We wish to thank Dr. NISHIJIMA for informing us of his results prior to publication.

[12] A special case of this formula is the commutation relation between the current and the incoming field operator which was derived in the same way by M. CINI and S. FUBINI: *Nuovo Cimento*, **2**, 860 (1955).

[13] Compare also S. SCHWEBER: *Nuovo Cimento*, **2**, 397 (1955).

This relation follows by applying (13) and (34):

$$[A_{\text{out}}(x)A_{\text{in}}(y)] - [A_{\text{in}}(x)A_{\text{in}}(y)] = \int \mathrm{d}x'\, \Delta(x - x')K_{x'}[A(x')A_{\text{in}}(y)] =$$

$$= -i \int \mathrm{d}x'\, \Delta(x-x')K_{x'} \int \mathrm{d}y'\, \Delta(y-y')K_{y'}R(x'; y') =$$

$$= -i \int \mathrm{d}x'\, \mathrm{d}y'\, \Delta(x - x')\Delta(y - y')K_x K_{y'}R(x'; y') \ (^{14}) .$$

By Fourier transformation (36) can also be written in the equivalent form

$$(36') \quad \begin{cases} [a_{\text{out}}(k)a^*_{\text{in}}(k')] = 2k_0\, \delta(\boldsymbol{k} - \boldsymbol{k}') - \\ \qquad\qquad - \dfrac{i}{(2\pi)^3} \displaystyle\int \mathrm{d}x\, \mathrm{d}x'\, \exp\,[i(k'x' - kx)]K_x K_{x'}R(x; x') , \\ [a_{\text{out}}(k)a_{\text{in}}(k')] = \dfrac{i}{(2\pi)^3} \displaystyle\int \mathrm{d}x\, \mathrm{d}x'\, \exp\,[-i(kx + k'x')]K_x K_{x'}R(x; x') . \end{cases}$$

The more general formula

$$(37) \quad [\ldots [a_{\text{out}}(k) - a_{\text{in}}(k), a^*_{\text{in}}\, k_1)] \ldots a^*_{\text{in}}\, k_i)] =$$

$$= -\frac{i}{(2\pi)^{\frac{3}{2}(n+1)}} \int \mathrm{d}x\, \mathrm{d}x_1 \ldots \mathrm{d}x_n \exp\,[i(\sum k_i x_i - kx)]K_{x_1} \ldots K_{x_n}R(x; x_1 \ldots x_n) ,$$

follows in the same way from (13) and (35).

3. – Simple applications of R-products.

As an example for the application of R-products we consider first the S-matrix element for the scattering of two bosons.

$$S(k', p'; k, p) = (\Phi^{k'p'}_{\text{out}}, \Phi^{kp}_{\text{in}}) .$$

The relation ($S = 1 - iT$)

$$(38) \quad T(k', p'; k, p) = \frac{1}{(2\pi)^3} \int \mathrm{d}x\, \mathrm{d}x'\, \exp\,[i(kx - k'x')]K_x K_{x'}(\Phi^{p'}_{\text{in}}, R(x'; x)\Phi^{p}_{\text{in}}) ,$$

---

(14) The interchange of the order of integration in this double integral can be justified with the aid of the commutators of the incoming and outgoing field. The following theorem can be proved: The order of integration in the above integral may be interchanged if and only if the incoming and outgoing fields satisfy the same commutation relations. The same applies to the order of integration in (35) and (37).

which forms the basis for derivations of Goldberger's dispersion [15] relations follows quite simply if we only assume that the field operator fulfills the asymptotic condition.

Since

$$\Phi^k_{\text{out}} = \Phi^k_{\text{in}} = \Phi^k ,$$

$$(\Phi^{k'p'}_{\text{out}}, \Phi^{kp}_{\text{in}}) = (\Phi^{p'}_{\text{out}}, a_{\text{out}}(k')a^*_{\text{in}}(k)\Phi^p_{\text{in}}) = (\Phi^{p'}, a_{\text{out}}(k')a^*_{\text{in}}(k)\Phi^p) ,$$

$$= (\Phi^{p'}, [a_{\text{out}}(k'), a^*_{\text{in}}(k)]\Phi^p) + 4k_0p_0\,\delta(\boldsymbol{k}'-\boldsymbol{p})\,\delta(\boldsymbol{p}'-\boldsymbol{k}) .$$

Insertion of equation (36′) for the commutator between outgoing and incoming fields

$$(\Phi^{p'}, [a_{\text{out}}(k'), a^*_{\text{in}}(k)]\Phi^p) = 4k_0p_0\,\delta(\boldsymbol{k}'-\boldsymbol{k})\,\delta(\boldsymbol{p}'-\boldsymbol{p}) -$$

$$- \frac{i}{(2\pi)^3}\int \mathrm{d}x\,\mathrm{d}x'\,\exp[i(kx-k'x')]K_xK_{x'}(\Phi^{p'}, R(x';x)\Phi^p) ,$$

yields the relation (38).

This expresses the scattering amplitude directly as a matrix element of $R(x';x)$ between one-particle states. If the S-matrix is causal, i. e. $A(x)$ satisfies the commutator condition, $(\Phi^{p'}, R(x';x)\Phi^p)$ is an invariant function that vanishes outside the forward cone of $x'-x$. This property, together with some consequences of the energy momentum spectrum, suffices in the case of forward scattering ($p'=p$, $k'=k$) to prove the dispersion relation for the forward amplitude and thereby obtain an explicit property of a causal S-matrix.

For non-forward scattering dispersion relations do not follow directly from (38). To obtain in this case enough consequences of causality for the scattering amplitude one has to reduce it to the vacuum expectation value of $R(x';x,y,y')$. This can be done as follows:

$$(\Phi^{p'}, R(x';x)\Phi^p) = (\Phi^{p'}, R(x';x)a^*_{\text{in}}(p)\Omega) =$$

$$= 2p_0\,\delta(\boldsymbol{p}-\boldsymbol{p}')(\Omega, R(x';x)\Omega) + (\Phi^{p'}, [R(x';x)a^*_{\text{in}}(p)]\Omega) ,$$

$$= 2p_0\,\delta(\boldsymbol{p}-\boldsymbol{p}')(\Omega, R(x';x)\Omega) + \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}y\,\exp[ipy]K_y(\Phi^{p'}, R(x';xy)\Omega) .$$

Again we have used the commutator (34′) of the R-product with the incoming

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[15] M. L. GOLDBERGER: *Phys. Rev.*, **99**, 979 (1955); R. OEHME: *Nuovo Cimento*, **4**, 1316 (1956); K. SYMANZIK: *Phys. Rev.*, **105**, 743 (1957).

field. In the same manner

$$(\Phi^{p'}, R(x' : xy)\Omega) = (\Omega, [a_{\text{in}}(p'), R(x'; x, y)]\Omega) =$$
$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\int \mathrm{d}y' \exp[-ip'y']K_{y'}(\Omega, R(x'; x, y, y')\Omega) .$$

Combining these equations we obtain the desired relation

$$(39) \qquad T(k', p'; kp) =$$
$$= \frac{1}{(2\pi)^6}\int \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}y\,\mathrm{d}y' \exp[i(kx + py - k'x' - p'y')]K_x K_{x'} K_y K_{y'} r(x'; x, y, y') .$$

According to Bogoljubow ([16]) dispersion relations for non-forward scattering can be derived from this equation if in addition to the commutator condition certain consequences of the energy-momentum spectrum are taken into account, which follow from the identity

$$(40) \qquad R(x'; xyy') = i\theta(x' - x)\{[R(x', yy'), A(x)] + [R(x'; y)R(x; y')] +$$
$$+ [R(x'; y')R(x; y)] + [A(x'), R(x; yy')]\} .$$

Turning to more general processes we wish to point out that all S-matrix elements with two incoming particles (this is not a serious restriction in view of experimental possibilities) and an arbitrary number of outgoing particles may be expressed by means of retarded products. Assuming invariance under space-time inversion, we have for $n \geqslant 3$ the relation

$$(41) \qquad S(p_1, \ldots, p_n; q_1, q_2) = S(q_1, q_2; p_1 \ldots p_n) = (\Phi_{\text{out}}^{q_1 q_2}, \Phi_{\text{in}}^{p_1 \ldots p_n}) =$$

$$(42) \qquad = \frac{-i}{(2\pi)^{\frac{3}{2}n}}\int \mathrm{d}x_2 \ldots \mathrm{d}x_n\,\mathrm{d}y_2 \exp[i(p_2x_2 + \ldots + p_nx_n) - iq_2y_2]\cdot$$
$$\cdot K_{x_2} \ldots K_{x_n}K_{y_2}(\Phi^{q_1}, R(y_2; x_2 \ldots x_n)\Phi^{p_1}) =$$

$$(43) \qquad = \frac{-i}{(2\pi)^{\frac{3}{2}(n+2)}}\int \mathrm{d}x_1 \ldots \mathrm{d}x_n\,\mathrm{d}y_1\,\mathrm{d}y_2 \exp[i(\sum_1^n p_ix_i - \sum_1^2 q_iy_i)]\cdot$$
$$\cdot K_{x_1} \ldots K_{x_n}K_{y_1}K_{y_2}r(y_2; y_1x_1 \ldots x_n) =$$

To derive (42), we note that an elementary calculation yields

$$(\Phi_{\text{out}}^{q_1q_2}, \Phi_{\text{in}}^{p_1 \ldots p_n}) = [\Phi^{q_1}, (\ldots[a_{\text{out}}(q_2) - a_{\text{in}}(q_2), a_{\text{in}}^*(p_2)] \ldots a_{\text{in}}^*(p_2))\Phi^{p_1}] .$$

([16]) N. N. Bogoljubow: *Report at the Int. Conference on Theor. Physics, Seattle,* 1956.

([17]) K. Symanzik: *Phys. Rev.*, **105**, 743 (1957).

In this formula we have only to insert (37) on the right-hand side to obtain (42). Eq. (43) follows easily if we apply the commutation relation (34′) twice to (42).

The equations treated in this section are examples of reduction formulae for retarded products. Matrix elements of R-products (and the S-matrix) are reduced to other matrix elements of R-products between states with a smaller number of particles; in particular to vacuum expectation values.

Another example is the relation

$$(44) \qquad (\Phi^p, A(x)\Phi^{p'}) = (\Omega, [a_{\text{in}}(p), A(x)]\Phi^{p'}) = \\ = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \mathrm{d}y \exp[-ipy] K_y(\Omega, R(x; y)\Phi_{p'}) ,$$

which has been used to discuss the analytic properties of the vertex function ([14]).

Very general reduction formulae follow if matrix elements between arbitrary incoming states are taken in eq. (35) and (37). We do not consider them here in detail; they are closely connected with expansions of $A(x)$ and $R(x; x_1 \ldots x_n)$ with respect to incoming fields which we shall discuss in a separate paper.

Appendix

In this paper we have disregarded the possible existence of stable bound states. However, it may be shown that the results derived in Sect. 2 and 3 are generally valid. The asymptotic condition

$$(7) \qquad \lim_{t \to \pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A^f_{\substack{\text{out} \\ \text{in}}}(t)\Psi) ,$$

remains unchanged even in theories with bound states ([18]). Only, in this case the incoming and outgoing operators do not give an irreducible representation ([19]) of the free field commutation relations (3); i.e. the orthogonal system (21) obtained by applying the operators A_{in}, A_{out} to the vacuum state is not a complete system. Instead we require merely that the field operators $A(x)$ form a complete operator ring which means that all states can be constructed by multiple application of $A(x)$ to the vacuum vector and superposition of the resulting states.

([18]) Compare K. Symanzik: *Phys. Rev.*, **105**, 743 (1957).

([19]) In LSZ I we postulated the irreducibility of A_{in}, A_{out} in connection with the asymptotic condition; compare footnote ([5]) of that paper.

In Sect. 2 we used only the limit relation (7) and the commutator (3) of the free fields but not the completeness of the system (21). Therefore we may take over the commutation relations (34) and (36) and the applications made in Sect. 3 without change.

If we study in particular the interaction of a nucleon field $\psi(x)$ with a meson field $A_\alpha(x)$, taking into account existing bound states, the well-known expression for the meson-nucleon scattering amplitude

$$T(k', \alpha', p'; k, \alpha, p) = \frac{-i}{(2\pi)^3}\int \mathrm{d}x\,\mathrm{d}x' \exp\left[i(kx - k'x')\right] \cdot K_x K_{x'} \theta(x' - x)(\Phi^{p'}_{\mathrm{in}}, [A_{\alpha'}(x') A_\alpha(x)]\Phi^{p}_{\mathrm{in}}) \,, \tag{45}$$

follows in the same manner as eq. (38).

RIASSUNTO (*)

Si discute il concetto di matrice causale di scattering ricorrendo a commutatori multipli ritardati di operatori di campo.

(*) *Traduzione a cura della Redazione.*

Operating Characteristics of a Gas-Liquid Bubble Chamber.

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(ricevuto il 18 Maggio 1957)

Summary. – Experiments have been performed to determine the effect of temperature on the sensitivity of a bubble chamber filled with various concentrations of methane gas in liquid propane. It has been found that as the concentration of methane is increased the operating pressure, for any given degree of sensitivity, encreases by about 0.3 atmosphere for every 1 °C decrease in temperature achieved,

1. – Introduction.

Bubble chambers using pure propane liquid have been employed for some time for the detection of ionizing particles. The operating temperature is in the region of 60 °C, the corresponding pressure being about 20 atm. For bubbles to grow continuously round the nucleation centres produced by the ionization, the surface tension must have a low enough value which in this case is achieved by operating at a temperature sufficiently near to the critical temperature of the liquid. Argan and Gigli [1] have been able to operate at room temperature a bubble chamber containing liquid propane, the necessary low surface tension value being attained by dissolving in the liquid a gas such as methane under sufficient pressure. The possibility of being able to dispense with the hot, thermostatically controlled jacket is attractive; moreover by dissolving gas in a liquid of high atomic number it might be possible to operate such a bubble chamber at a much lower, and more conveniently realizable, temperature than that required for a conventional bubble chamber containing

[1] P. E. Argan and A. Gigli: *Nuovo Cimento*, **4**, 953 (1956).

only the pure liquid. Apart from xenon, which is prohibitively expensive, the only dense liquids containing atoms of high atomic number which are suitable as bubble chamber media have rather high operating temperatures, e.g., tungsten hexafluoride which becomes radiation-sensitive at 136 °C [2].

Temperature, however, is not the only important operating factor. The value of the pressure required initially to prevent the liquid from boiling, or releasing gas from solution, is also of major importance since, in particular, the glass windows must safely withstand this pressure. It is therefore of prime importance to see whether an unduly higher operating pressure accompanies the reduction in operating temperature achieved by dissolving gas in the pure liquid bubble chamber medium, and for this purpose we have determined the operating conditions, of temperature and pressure for a series of solutions of gaseous methane in liquid propane.

2. – Experimental method.

The chamber is constructed of brass and consists of a cylindrical volume, diameter 2 in., length 6 in., with 1 in. thick glass windows. Expansion is achieved by movement of an aluminium piston, which contains a small permanent magnet, in a cylinder of diameter $1\frac{1}{2}$ in. The magnet enables the position of the piston to be located during the filling process. Fast expansion and recompression are achieved by the usual method of controlling the gas pressure behind the piston with a solenoid valve, the photographs being taken using a flash of duration 2 μs. The chamber and expansion cylinder are well lagged, the temperature being controlled at the required value by regulating the power supplied to electric tape heaters wound round the chamber.

Commencing with pure propane, the temperature of the chamber was raised by stages until photographs taken with a ^{60}Co source near the chamber showed first random bubbles, then short tracks, long tracks, and finally, at the highest temperature, foam. The corresponding temperatures were then obtained after a little methane gas had been dissolved in the liquid propane, these temperatures being lower for each stage of sensitivity. Further experiments were carried out with increasing amounts of methane gas dissolved in the liquid propane until finally the chamber became sensitive at room temperature. The equilibrium total pressure of the solution in the chamber at any temperature was found by measuring the external pressure at which gas was first liberated from solution. Typical photographs are shown in Fig. 1 and Fig, 2.

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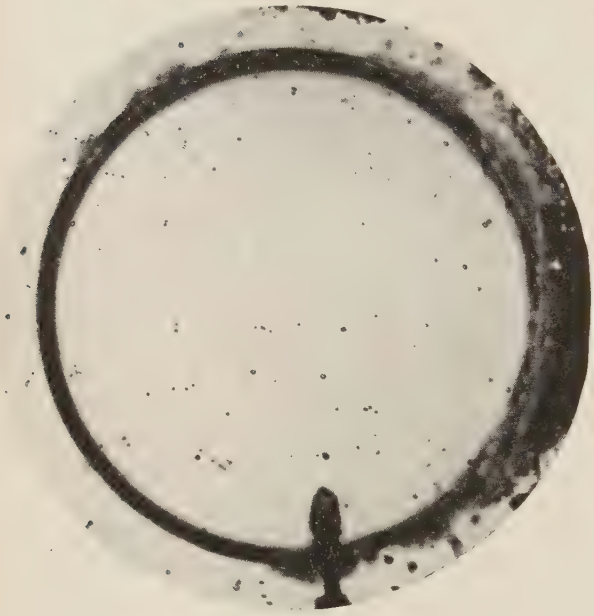
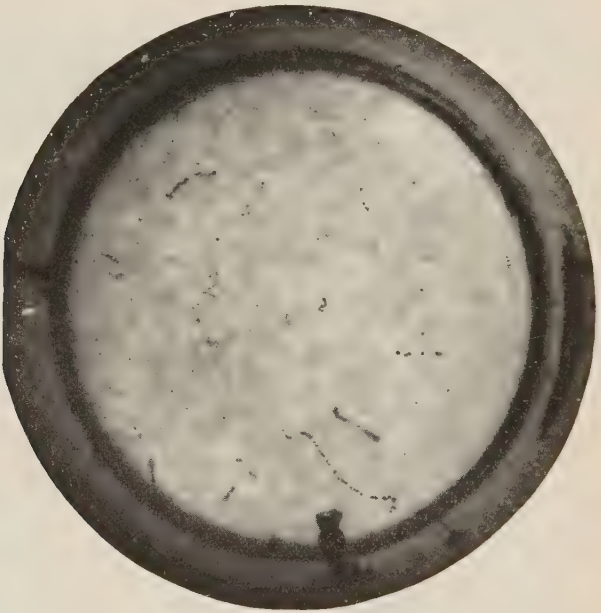
[2] J. H. Mullins, E. D. Alyea, L. R. Gallacher, J. K. Chang and J. M. Teem: *Bull. Amer. Phys. Soc.*, Series II, Vol. 2 (1957).



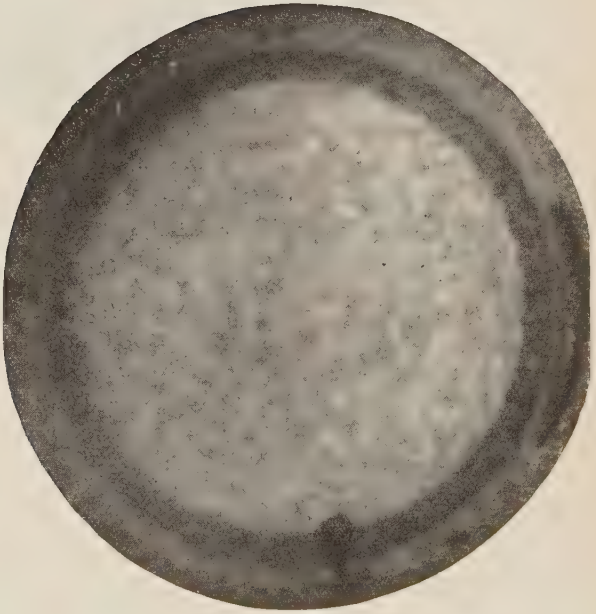
(*a*) random bubbles.

(*b*) definite tracks.

Fig. 1 *a*, *b*). – Typical photographs showing the various degrees of sensitivity of a methane in liquid propane bubble chamber.



(*c*) tracks on a foamy background.



(*d*) foam. The photographs were taken about 7 ms after the beginning of the expansion with a ^{60}Co source near the chamber.

Fig. 1 *c*, *d*). – Typical photographs showing the various degrees of sensitivity of a methane in liquid propane bubble chamber.

Fig. 2. – Tracks in liquid propane at 64.5 °C. The shorter tracks are due to an external ^{60}Co source. The longer tracks, passing right through the chamber, are due to a cosmic ray shower originating in the chamber wall at a point towards the top left corner of the photograph. The light flash, of duration 2 μs, occurred 9 ms after the beginning of the expansion.

3. – Results.

In Fig. 3 the temperature and pressure defining the region within which definite tracks are obtainable in a methane/propane solution are shown. If the prevailing conditions of temperature and pressure fall above the upper line, foam will result, while if they fall below the lower line, no tracks will be formed. Using the data of WEINAUG and KATZ [3] we have calculated that on Glaser's theory [4] the number of charge units produced per bubble by the source is two along the foam limit line and about six for the line giving the lower limit of sensitivity.

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[3] C. F. WEINAUG and D. L. KATZ: *Ind. Eng. Chem.*, **35**, 239 (1943).
[4] D. A. GLASER: *Suppl. Nuovo Cimento* , **11** 361 (1954(.

It is obvious from Fig. 3 that by adding the gas to the liquid the operating temperature is lowered but that the operating pressure is considerably increased.

If one considers this pressure to be the sum of the partial pressure of the dissolved gas and the vapour pressure of the liquid propane at the appropriate temperature, it follows that although the vapour pressure of the propane has been *reduced* by the lowering of the operating temperature, a much greater *increase* results from the pressure of the dissolved gas. The conclusion is therefore that in this case raising the temperature of the liquid is a more effective method of lowering its surface tension than dissolving gas in the liquid, and this seems also to be true for CO_2 gas dissolved in propane from the few experiments we have carried out on such a system. In general one would therefore only consider using a gas-in-liquid medium for a bubble chamber in those exceptional cases, if any, where the presence of a small partial pressure of gas considerably lowers the surface tension of the liquid, for only then will both the operating temperature and the pressure be lower than the values for operation with the pure liquid. If however the pure liquid is known to dissociate at a certain temperature, it would be possible to operate at a lower temperature at which no such dissociation occurs by using a suitable gas dissolved in the liquid as the bubble chamber medium, provided that the operating pressure is not thereby raised to a prohibitively high value.

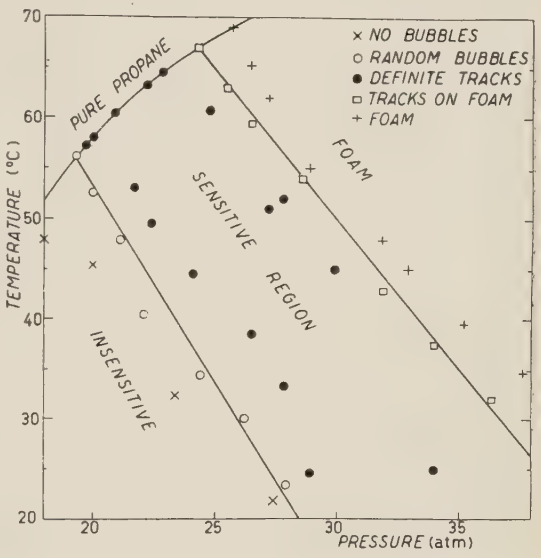

Fig. 3. – Diagram showing the effect of temperature on the sensitivity of a bubble chamber filled with various solutions of methane gas in liquid propane. Definite tracks are only obtainable within the region indicated.

RIASSUNTO (*)

Si sono eseguiti esperimenti per determinare l'effetto della temperatura sulla sensibilità di una camera a bolla riempita con differenti concentrazioni di metano gassoso in propano liquido. Si è trovato che aumentando la concentrazione del metano la pressione d'esercizio, per qualsiasi grado di sensibilità aumenta di circa 0.3 atmosfere per ogni °C di diminuizione della temperatura.

(*) *Traduzione a cura della Redazione.*

The High Energy Limit of the Potential Scattering (*).

I. - Non Relativistic Kinematics

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(ricevuto il 31 Maggio 1957)

Summary. — Some asymptotic expansions of the phase shifts have been derived, corresponding to high energies and a non relativistic potential scattering. They are valid for small as well as large values of the angular momenta and are obtained using the Gel'fand and Levitan operator which gives a uniform connection of two complete bases in the Hilbert space. An approximation of the total scattering amplitude is also presented, whose validity extends to all angles of scattering with the exception of the vicinity of 180°. Some remarks are made on the problem of obtaining the potential from the knowledge of the phase shifts for a fixed energy as function of the impact parameter.

Introduction.

The large number of experimental data on nuclear scattering at high energy raises once more the classical problem of the interpretation of the diffraction experiments in terms of the nature of the scatterer.

Since at high energy many angular momenta participate to the collision, an approximation of the total scattering amplitude, rather than of the partial waves, is needed. Some valuable approaches to this problem have been made recently (1). With them, the total scattering amplitude is expressed as an

(*) This paper was presented at the « Congresso Nazionale della Società Italiana di Fisica », held in Turin on September 6th, 1956. It was supported in part by the American Office of Naval Research.

(1) L. I, Schiff: *Phys. Rev.*, **103**, 443 (1956).

integral over the domain of the scatterer, which is not reduced further to simple analytical functions. This is indeed a difficult task since the behaviour of the total scattering amplitude as function of energy and angles of scattering is not simple because of the familiar diffraction patterns which must appear in it. From the other side the partial scattering phase shifts corresponding to a given impact parameter are expected to be smooth functions of the energy. This suggests the convenience of studying the asymptotic behaviour at high energy of these quantities. The problem of summing the partial waves amplitudes which becomes cumbersome when a large number of partial waves has to be considered, can also be worked out as it will be shown in Sect. 3.

Some asymptotic expansions of the scattering phase shifts valid for small angular momenta only, have been derived in an earlier paper [2]. With the exception of the S-wave, it was not possible there to give a compact expression for the general term of the expansion. The first two terms of it were obtained by evaluating asymptotically for large values of the energy the integrals of the perturbation theory. This is, however, a difficult procedure which fails completely when large impact parameters have to be considered.

We will present here a new approach to the problem, that avoids the perturbation expansion and allows approximations valid for small as well as large impact parameters.

In the old formulation use has been made of the familiar integral operator which links the base $\Phi_0(E, l, x)$ in the Hilbert space without interaction, with the base $\Phi(E, l, x)$ which diagonalizes the total hamiltonian with interaction. This can be expressed by the equation

$$\int_0^\infty \mathrm{d}x' \{G_0(E, l; x, x') - \delta(x - x')\} \Phi(E, l, x') = \Phi_0(E, l, x) ,$$

where $G_0(E, l; x, x')$ is the appropriate Green's function corresponding to the free hamiltonian.

The difficulty of obtaining the behaviour of Φ for large E from the above transformation that we shall briefly write in the form

$$\Omega_0 \Phi = \Phi_0$$

is due to the energy dependence of Ω_0.

A different connection between the two bases Φ and Φ_0 is possible. Choosing all Φ and Φ_0 so that for $x = 0$ they have the same behaviour independently

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[2] M. VERDE: *Nuovo Cimento*, **2**, 1001 (1955).

from the energy, a new linear transformation

$$\Phi = \Omega\Phi_0$$

exists with an integral operator Ω which does not contain E.

This transformation is given by the so called Gel'fand and Levitan's equation, which will form here the basis for the study of the behaviour of Φ when E is large.

The collision corresponding to small impact parameters b is studied in the first section and its generalization to large values of b is presented in Sect. **2**. The problem of summing over the partial waves amplitudes is treated in the Sect. **3** and **4**. Some remarks are made in the last section on the problem of obtaining the potential when all scattering amplitudes at a fixed energy, which for convenience can be chosen large enough, are known as functions of the impact parameter.

1. – Phase shifts for small angular momenta and large energies.

The Schrödinger equation for the partial wave of angular momentum l

$$\varphi'' - \left(V + \frac{l(l+1)}{r^2}\right)\varphi = -E\varphi\,, \tag{1.1}$$

can be written in an equivalent integral form as derived by Gel'fand and Levitan [1]

$$\Phi(E, x) = \Phi_0(E, x) + \int_0^x \mathrm{d}y K(x, y)\Phi_0(E, y)\,. \tag{1.2}$$

$\Phi(E, x)$ is a solution of (1.1) normalized so that for $r = 0$ it behaves as r^{l+1} for all values of the eigenvalue E in the discrete spectrum as well as in the continuum. $\Phi_0(E, x)$ are the solutions, normalized in the same way, of the equation corresponding to the free waves:

$$\varphi_0'' - \frac{l(l+1)}{r^2}\varphi_0 = -E\varphi_0\,. \tag{1.1'}$$

The kernel $K(x, y)$ of the Eq. (1.2) can be expressed in terms of $\Phi(E, x)$ and $\Phi_0(E, x)$. For this purpose the so called spectral functions $\varrho(E)$ and $\varrho_0(E)$ which enter in the completness relations:

$$\int_{-\infty}^{+\infty} \mathrm{d}\varrho(E)\Phi(E, x)\Phi(E, x') = \delta(x - x') = \int_{-\infty}^{+\infty} \mathrm{d}\varrho_0(E)\Phi_0(E, x)\Phi_0(E, x')\,,$$

are needed. The kernel $K(x, y)$ is given by

$$K(x, y) = -\int_{-\infty}^{+\infty} \mathrm{d}\sigma(E)\Phi(E, x)\Phi_0(E, y)\,, \tag{1.3}$$

$$\mathrm{d}\sigma(E) = \mathrm{d}[\varrho(E) - \varrho_0(E)]\,.$$

Some values of $K(x, y)$ are [3]

$$\begin{cases} K(x, 0) = 0\,, \\ K(x, x) = -\dfrac{1}{2}\displaystyle\int_0^x V(x')\,\mathrm{d}x'\,. \end{cases} \tag{1.4}$$

The spectral function $\varrho(E)$ is related to the solutions of Eq. (1.1) as follows:

$$\begin{cases} E = -\infty & \varrho(-\infty) = 0\,, \\ E < 0 & \dfrac{\mathrm{d}\varrho}{\mathrm{d}E} = \displaystyle\sum_n \delta(E - E_n)\,\frac{1}{\int_0^\infty \Phi^2(E_n, x)\,\mathrm{d}x}\,, \\ E > 0 & \dfrac{\mathrm{d}\varrho}{\mathrm{d}t} = \dfrac{k}{\pi}\,\dfrac{1}{|f(k)|^2}\,, \end{cases} \tag{1.5}$$

where E_n, are the energies corresponding to the bound states and $f(k)$ is connected to the solution $f(k, r)$ of Eq. (1.1) which for large r behaves as

$$f(k, r) \xrightarrow[r\to\infty]{} \exp[ikr]\,. \tag{1.6}$$

It is

$$f(k) = \lim_{r\to 0} (2l + 1)r^l f(k, r)\,. \tag{1.7}$$

From the corresponding definition of $\varrho_0(E)$ one obtains the following values:

$$\begin{cases} E < 0 & \dfrac{\mathrm{d}\varrho_0}{\mathrm{d}E} = 0\,, \\ E > 0 & \dfrac{\mathrm{d}\varrho_0}{\mathrm{d}E} = \dfrac{1}{2}\left(\dfrac{2}{k}\right)^{2l-1}\left[\dfrac{\Gamma(\frac{1}{2} - l)}{\Gamma(\frac{1}{2} + l)}\right]^2\,. \end{cases} \tag{1.8}$$

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[3] Gel'fand and Levitan: *Isvestia Akad. Nauk SSJR.* **15**, 309 (1951); R. Jots and W. Kolm: *Det. Kong. Danske Videns. Selskab*, **27**, 9 (1953).

Substituting Eq. (1.3) in Eq. (1.2) we get

$$\Phi(E, x) = \Phi_0(E, x) - \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E')\Phi(E', x) \int_0^x \mathrm{d}y \Phi_0(E'y)\Phi_0(Ey) ,$$

which can also be written

$$(1.9) \qquad \Phi(E, x) = \Phi_0(E, x)\left[1 + \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0'(E', x)}{E' - E}\right] -$$

$$- \Phi_0'(E, x) \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0(E', x)}{E' - E} ,$$

since

$$(E' - E) \int_0^x \Phi_0(E', y)\Phi_0(E, y)\,\mathrm{d}y = \Phi_0'(E, x)\Phi_0(E', x) - \Phi_0(E, x)\Phi_0'(E', x) ,$$

Φ_0' is the derivative of Φ_0 with respect to x.

Putting:

$$(1.10) \qquad \frac{1}{k} \operatorname{tg} \delta_l(E, x) =$$

$$= - \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0(E', x)}{E' - E} \Big/ \left\{1 + \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0'(E', x)}{E' - E}\right\} .$$

From Eq. (1.9) one deduces for the phase shifts $\delta_l(E)$ the following relation:

$$(1.11) \qquad \delta_l(E) = \delta_l(E, \infty) .$$

In order to establish expansions of tg δ_l for large E, we have to study the behaviour of the two functions:

$$(1.12) \qquad \begin{cases} K(E, x) = \displaystyle\int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0(E', x)}{E' - E} , \\ H(E, x) = \displaystyle\int_{-\infty}^{+\infty} \mathrm{d}\sigma(E') \frac{\Phi(E', x)\Phi_0'(E', x)}{E' - E} , \end{cases}$$

for large E and $x = \infty$. Admitting the existence of an expansion of K and H

in inverse powers of E, one obtains:

$$K(E, x) = -\frac{1}{E}\sum_0^\infty{}_n \frac{1}{E^n} K_n(x)\,, \qquad H(E, x) = -\frac{1}{E}\sum_0^\infty{}_n \frac{1}{E^n} H_n(x)\,,$$

where

$$\text{(1.13)} \qquad \begin{cases} K_n(x) = \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E')(E')^n \Phi(E', x)\Phi_0(E', x)\,, \\ H_n(x) = \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E')(E')^n \Phi(E', x)\Phi_0'(E', x)\,. \end{cases}$$

From Eqs. (1.11), (1.10) and (1.12)

$$\text{(1.14)} \qquad \frac{1}{k}\operatorname{tg}\delta_l = \frac{-K(E, \infty)}{1 + H(E, \infty)} = \sum_0^\infty{}_n \frac{K_n(\infty)}{E^{n+1}} \Big/ \Big(1 - \sum_0^\infty{}_n \frac{1}{E^{n+1}} H_n(\infty)\Big)\,.$$

The moments $K_n(x)$ and $H_n(x)$ defined by Eq. (1.13) satisfy to simple recurrence equations that serve for their evaluation in terms of the potential $V(x)$. Indeed the first moment

$$K_0(x) = \int_{-\infty}^{+\infty} \mathrm{d}\sigma(E')\Phi(E', x)\Phi_0(E', x)\,,$$

coincides with $K(x, x)$ of Eq. (1.3) and becomes after Eq. (1.4)

$$K_0(x) = -\frac{1}{2}\int_0^x V(x')\,\mathrm{d}x'\,,$$

with the definition of K_n and H_n given in Eq. (1.13) it is furthermore easy to see that:

$$\text{(1.15)} \qquad \begin{cases} K_{n+1}'(x) = -\tfrac{1}{2}(H_n'' - VH_n') + \dfrac{l(l+1)}{x}\dfrac{\mathrm{d}}{\mathrm{d}x}\Big(\dfrac{K_n(x)}{x}\Big)\,, \\ H_n'(x) \quad = \tfrac{1}{2}(K_n'' - VK_n)\,. \end{cases}$$

From Eq. (1.9) in order to satisfy the boundary condition,

$$H(E, x) - (l+1)K(E, x)/x$$

must vanish for $x = 0$. This means

$$\begin{cases} K(E, 0) = 0 , \\ (l+1)\, K'(E, 0) = H(E, 0) , \end{cases} \tag{1.16}$$

there is $K_n(0) = 0$, $H_n(0) = (l+1) K_n'(0)$.

The solution of Eq. (1.15) with the above boundary conditions is:

$$\begin{cases} K_0(x) = -\frac{1}{2}\int_0^x V(y)\,\mathrm{d}y , \\ H_n(x) = \frac{1}{2} K_n'(x) + K_n(x) K_0(x) - \int_0^x K_n'(y) K_0(y)\,\mathrm{d}y + \left(l + \frac{1}{2}\right) K_n'(0), \\ K_{n+1}(x) = \frac{1}{2}\left(H_n'(0) - H_n'(x)\right) - K_0(x) H_n(x) + \int_0^x K_0(y) H_n'(y)\,\mathrm{d}y + \\ \qquad + \frac{l(l+1)}{2}\left[\int_0^x \frac{K_n''(y)}{y}\,\mathrm{d}y - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{K_n(x)}{x}\right)\right]. \end{cases} \tag{1.17}$$

Therefore for the first three terms of the expansion of the phase shift itself we get

$$\delta_l = -\frac{1}{2k}\int_0^\infty V\,\mathrm{d}x - \\ -\left(\frac{1}{2k}\right)^3\left[V'(0) + \int_0^\infty V^2\,\mathrm{d}x\right] - \frac{(l+\frac{1}{2})^2 - \frac{1}{4}}{4k^3}\int_0^\infty \frac{V'(x)}{x}\,\mathrm{d}x + o\,(1/k^5) . \tag{1.18}$$

This expansion is meaningless if the coefficients which appear in it become infinite. This is f. i. the case if $V'(x)$ does not vanish for $x = 0$. We also remark that in the successive terms, higher powers of l appear so that the vanishing at the origin of derivatives of V of higher order is requested.

Eq. (1.18) is useless if l becomes large. This case will therefore be considered in the next section. For a potential even in x, $K_0(x)$ is odd and all other K_n are also odd in x as it can be seen from Eq. (1.17). This is sufficient for the existence of $K_n(\infty)$ and $H_n(\infty)$ for every n. If $K_0(x)$ is odd up to a term $2n+1$, the expansion becomes meaningless only after the term of order n.

2. – Phase shifts for large impact parameters.

The difficulty of obtaining expansions of the phase shifts for large impact parameters and large energies is already present for the free scattering, where an approximation of the Bessel functions of large order and argument is needed.

This can be achieved by a transformation of Eq. (1.1)′ into an equation corresponding to a motion with a fixed angular momentum.

One has to introduce [1] a new independent variable $q(x)$ and a new function $\psi_0(q)$ instead of x and $\Phi_0(x)$, by means of the definition

$$\Phi_0(x) = \frac{1}{\sqrt{q'(x)}}\,\psi_0(q)\,. \tag{2.1}$$

Eq. (1.1)′ become then

$$\frac{\mathrm{d}^2\psi_0}{\mathrm{d}q^2} + \left(k^2 + \frac{5}{36q^2}\right)\psi_0 = 0\,, \tag{2.2}$$

if $q(x)$ satisfy the non-linear differential Eq.

$$\left(k^2 + \frac{5}{36}\,\frac{1}{q^2}\right)q'^2 - \frac{3}{4}\left(\frac{q''}{q'}\right)^2 + \frac{1}{2}\,\frac{q'''}{q'} = k^2 - \frac{b^2 - \frac{1}{4}}{x^2}\,, \tag{2.3}$$

where $q'(x) = \mathrm{d}q/\mathrm{d}x$ and $b = l + \frac{1}{2}$.

A solution of Eq. (2.3) can be found for large values of b. Indeed introducing the new independent variable

$$z = \left\{\left(\frac{kx}{b}\right)^2 - 1\right\}^{\frac{1}{2}}\,,$$

an expansion of q in inverse powers of b^2 can be carried out and the coefficients are elementary functions if expressed in the variable z. For the first two terms one obtains

$$\frac{k}{b}\,q(z) = (z - \operatorname{arctg} z) + \frac{1}{b^2}\left(\frac{5}{72}\,\frac{1}{z - \operatorname{artg} z} - \frac{5}{24}\left(\frac{1}{z}\right)^3 - \frac{1}{8z}\right) + \cdots\,, \tag{2.4}$$

we remark here that near the turning point $x = b/k$ t.i. $z = 0$

$$\frac{k}{b}\,q(z) \simeq -\frac{z}{70b^2}\,. \tag{2.5}$$

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[1] This transformation has been extensively studied recently by some mathematicians. See f.i. CHERRY: *Trans. Amer. Math. Soc.*, **68**, 224 (1950).

Considering however the first term only of the expansion, we would have obtained instead

$$\frac{k}{b}\, q(z) \simeq \frac{1}{3}\, z^3 .$$

By means of Eq. (2.1) the needed asymptotic expansion of the Bessel functions are deduced.

We can now perform the same transformation (2.1) in Eq. (1.1), with the result

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}q^2} + \left(k^2 + \frac{5}{36q^2} + \frac{U(x(q))}{q_x'^2}\right)\psi = 0 . \tag{2.6}$$

All considerations made in Sect. **1** are valid for our transformed equations (2.3) and (2.6) instead of Eq. (1.2)′ and (1.1). In this second case $(l+\frac{1}{2})^2 = \frac{1}{3}$ and the potential V as function and the variable q is defined by

$$V(q) = U(x(q))\left(\frac{\mathrm{d}x}{\mathrm{d}q}\right)^2 . \tag{2.7}$$

Since in Eq. (1.18) the square of $l+\frac{1}{2}$ appears we have not to pay attention to the particular choice of solutions of (2.6) in order to satisfy the boundary condition at $x=0$. This means that Eq. (1.18) can be retained in our case with $(l+\frac{1}{2})^2 = \frac{1}{9}$ and $V(q)$ instead of $V(x)$. Therefore

$$\delta_l = -\frac{1}{2k}\int_0^\infty V(q)\,\mathrm{d}q - \left(\frac{1}{2k}\right)^3\left[\int_0^\infty V^2(q)\,\mathrm{d}q + \frac{1}{6}\int_0^\infty \frac{V'(q)}{q}\,\mathrm{d}q\right] + 0(1/k^5) . \tag{2.8}$$

We wish to remark here that all integrals exist, since $V(q)$ like $U(x)$ is rapidly vanishing for large q, and that for $q \to 0$, $V(q) \cong q^2$ as it can easily be verified by means of Eqs. (2.7) and (2.4). This is also the reason why the term $V'(0)$ of Eq. (1.11) does not appear above.

The dependence of the phase shift on l is contained in the function q as given in Eq. (2.4). The asymptotic expansion (2.8) is not a simple expansion in inverse powers of k since k appears also in q. The first term of (2.8) reduces to a well known result [5]. We may indeed write

$$\int_0^\infty U(q)\,\mathrm{d}q = \int_0^\infty U\left(\frac{b}{k}\sqrt{z^2+1}\right) G(b,z)\,\mathrm{d}\left(\frac{bz}{k}\right),$$

[5] G. Molière: *Zeits. f. Naturfor.*, **2**, 183 (1947).

where

$$G(b, z) = \frac{z^2}{z^2 + 1} \frac{1}{k/b(q'(z))},$$

for

$$b = \infty \qquad G\,b,z) = 1,$$

and

$$\delta = -\frac{1}{2k}\int_0^\infty U(\sqrt{x^2 + b^2}/k^2)\,\mathrm{d}x, \tag{2.9}$$

for b finite but large, $G(b, z)$ is zero for $z = 0$ but increases rapidly and reaches oscillating the value one. This has to be interpreted as due to the physical reason that outer regions of the scatterer are involved for large impact parameters.

With the assumption $G(b, z) = 1$, the second term in (2.8) becomes meaningless. For its existence it is necessary to keep the second term in Eq. (2.4).

The expansion (2.8) is very convenient for numerical calculations of phase shifts, since $q(z)$ is an universal function, independent from the choice of the particular potential, and the numerical integrations in Eq. (2.8) can be easily performed. The most important advantage of Eq. (2.8) consists perhaps in showing how a potential of given shape influences the values of the phase shifts. A small variation ΔV of the shape of V brings in the small variation,

$$\Delta\delta_l = -\frac{1}{2k}\int_0^\infty (\Delta V)\,\mathrm{d}q - \left(\frac{1}{2k}\right)^3 \left[2\int_0^\infty V(\Delta V)\,\mathrm{d}q + \frac{1}{6}\int_0^\infty \frac{(\Delta V)'\,\mathrm{d}q}{q}\right] + 0(1/k^5).$$

3. – Approximation for the eigenfunctions of large angular momenta.

In order to sum over the scattering amplitudes of the partial waves corresponding to a given angular momentum, an expansion of the Legendre polynomial valid for large values of l is needed. The same method developed before will be used here. Indeed the function $\chi = \sqrt{\sin\theta}\, P_l(\cos\theta)$ is the regular solution of the differential equation

$$\frac{1}{\chi}\frac{\mathrm{d}^2\chi}{\mathrm{d}\theta^2} + \left(b^2 + \frac{1}{4\sin^2\theta}\right) = 0, \tag{3.1}$$

where $b = l + \frac{1}{2}$.

We assume the equation of the Bessel function of order zero

$$\frac{1}{\chi_0}\frac{\mathrm{d}^2\chi_0}{\mathrm{d}\theta^2}+\left(b^2+\frac{1}{4\theta^2}\right)=0\,, \tag{3.2}$$

as the corresponding in our case of Eq. (1.1)′. Then (3.1) is equivalent to the Gel′fand and Levitan integral equation

$$\chi(b,\theta)=\sqrt{\theta}J_0(b\theta)+\int_0^\theta K(\theta,t)\sqrt{t}J_0(bt)\,\mathrm{d}t\,. \tag{3.3}$$

Where $K(\theta, t)$ is the kernel defined by Eq. (1.3). In particular from Eq. (1.4)

$$K(\theta,\theta)=-\frac{1}{2}\int_0^\theta\frac{1}{4}\left(\frac{1}{\sin^2 t}-\frac{1}{t^2}\right)\mathrm{d}t\,.$$

Since we are interested in the asymptotic behaviour of $\chi(b, \theta)$ itself for large b, it is more convenient to perform successive partial integrations in Eq. (3.3). These give (*)

$$\chi(b,\theta)=\sum_0^\infty{}_n\left(\frac{1}{b}\right)^n S_n(\theta)\theta^{n+\frac{1}{2}}J_n(b\theta)\,, \tag{3.4}$$

where $S_n(\theta)$ is independent of b and it is defined by

$$\begin{cases} S_0(\theta)=1\,, \\ S_n(\theta)=\dfrac{1}{\sqrt{\theta}}\left(\dfrac{1}{t}\dfrac{\partial}{\partial t}\right)^{n-1}\left(\dfrac{K(\theta,t)}{\sqrt{t}}\right)\Big|_{t=\theta}\,. \end{cases} \tag{3.5}$$

There is a simple connection between the derivatives

$$\left(\frac{\partial}{\partial t}\right)^n K(\theta,t)\Big|_{t=\theta}\,,$$

and the moments K_n and H_n defined in Eq. (1.13). We put:

$$K_n(\theta,t)=-\int_0^{+\infty}\mathrm{d}\sigma(b^2)\chi(b^2,\theta)\chi_0(b^2,t)b^{2n}\,,$$

$$H_n(\theta,t)=-\int_0^{\infty}\mathrm{d}\sigma(b^2)\chi(b^2,\theta)\chi_0'(b^2,t)b^{2n}\,,$$

(*) In the mathematical literature one finds the formula Eq. (3.4) established by a different method which in particular does not give the general expression Eq. (3.5). See G. Szego: *Proc. London Math, Soc.*, (2), **36**, 427 (1934).

then $K_n(\theta, \theta)$ and $H(\theta, \theta)$ are the moments corresponding to Eq. (1.13). Since

$$b^2\chi_0(b^2, t) = -\left(\frac{\partial^2}{\partial t^2} + \frac{1}{4t^2}\right)\chi_0(b^2, t)\,,$$

one obtains

$$K_n(\theta, t) = (-1)^n\left(\frac{\partial^2}{\partial t^2} + \frac{1}{4t^2}\right)^n K_n(\theta, t)\,,$$

$$H_n(\theta, t) = (-1)^n\left(\frac{\partial^2}{\partial t^2} + \frac{1}{4t^2}\right)^n H_n(\theta, t)\,.$$

These relations allow the evaluation of all derivatives $(\partial^n/\partial t^n)K(\theta, t)|_{t=0}$ in terms of the moments $K_n(\theta)$ and $H_n(\theta)$.

By means of the recurrence relations Eq. (1.17) the general term $S_n(\theta)$ can be calculated. In particular:

$$S_1(\theta) = \frac{1}{8}\left(\operatorname{cotg}\theta - \frac{1}{\theta}\right),$$

$$S_2(\theta) = \frac{1}{128\theta^2}\left\{-15 + 6\theta\operatorname{cotg}\theta + 8\theta^2 + 9\theta^2\operatorname{cotg}^2\theta\right\}.$$

The expansion (3.4) is valid for all θ with the exception of $\theta = 180°$ because at this angle the $S_n(\theta)$ diverge. From Eq. (3.4) it is also clear that the successive terms of the expansions represent corrections for large angles only.

4. – The total scattering amplitude.

When the number of phase shifts becomes large it is cumbersome to perform the sum of the partial waves in order to obtain the total scattering amplitude. In this case however the substitution of the sum with an integration over the impact parameter which is treated as a continuous variable is appropriate. We write:

$$f(\theta) = \frac{1}{k}\sum(2l+1)\exp[i\delta_l]\sin\delta_l P_l(\cos\theta) = \frac{2}{k}\int_0^\infty b\,\mathrm{d}b\,\exp[i\delta_b]\sin\delta_b P_b(\cos\theta)\,,$$

with the usual notation for $b = l + \frac{1}{2}$. Using Eq. (3.4)

$$\sqrt{\sin\theta}f(\theta) = \frac{2}{k}\sum_0^\infty{}_n\, S_n(\theta)\theta^{n+\frac{1}{2}}\int_0^\infty b^{-n+1}\,\mathrm{d}b\,\exp[i\delta_b]\sin\delta_b J_n(b\theta)\,,$$

for a better approximation of course, one should keep for the first phase shifts the rigorous formula. Calling $M_n(k, \theta)$ the following moments

$$M_n(k, \theta) = \int_0^\infty b^{-n+1}\, \mathrm{d}b \exp[i\delta_c] \sin \delta_c J_n(b\theta)\,, \tag{4.1}$$

it is

$$f(\theta) = \frac{2}{k}\sqrt{\frac{\theta}{\sin\theta}} \sum_0^\infty{}_n S_n(\theta)\theta^n M_n(k, \theta). \tag{4.2}$$

For $\theta = 0$ only the first moment is involved

$$f(0) = \frac{2}{k} M_0(k, 0) = \frac{2}{k}\int_0^\infty b\, \mathrm{d}b \exp[i\delta_0] \sin \delta_0\,.$$

The total cross-section becomes

$$\sigma = \frac{4\pi}{k} \operatorname{Im} f(0) = \frac{8\pi}{k^2}\int_0^\infty b\, \mathrm{d}b \sin^2 \delta_b\,.$$

The successive terms in the expansion (4.2) are corrections for large angles of scattering. The evaluation of (4.1) needs the knowledge of the phase shifts for large as well as for small values of b. This knowledge has been established by means of the approximations Eqs. (1.18) and (2.8) valid for large energies.

As an illustration we wish to find the values of the moments (4.1) in the simple case of a scattering amplitude $t = \exp[i\delta_b] \sin \delta_b$ constant in the domain R of the scatterer and zero outside. They are

$$\begin{cases} M_n(k, \theta) = t\int_0^{kR} b^{-n+1} J_n(b\theta)\, \mathrm{d}b = -\dfrac{t}{\theta}\left\{(kR)^{-n+1} J_{n-1}(kR\theta) - \left(\dfrac{\theta}{2}\right)^{n-1} \dfrac{1}{(n-1)!}\right\}, \\ M_0(k, \theta) = t\,\dfrac{kR}{\theta}\, J_1(kR\theta)\,. \end{cases}$$

This particular case clearly indicates the presence of the familiar diffraction patterns in the scattering and may serve as an indication of the contribution coming from terms of higher order for different values of kR.

5. – The inversion problem.

An interesting question which arises in scattering problems is to give the potential, once all phase shifts are assigned as function of the impact parameter for a fixed value of the energy.

It is hard to believe that an answer to this problem can be found for all values of the energy. This is because the functional dependence of the scattering phase shifts on the potential is very complicated. If, however, the energy is chosen large enough so that the first term only of the expansion (2.8) is important, the functional dependence of the phase shifts on the potential becomes linear. The problem in this case is reduced to the solution of a linear integral equation. Indeed Eq. (2.9)

$$-2k\delta = \int_0^\infty U(\sqrt{x^2 + b^2/k^2})\,\mathrm{d}x\,,$$

can easely be inverted. The inversion formula reads:

$$rU(r) = -\frac{4k}{\pi}\int_0^\infty\int_0^\infty \mathrm{d}\varrho\,\mathrm{d}\omega\,\varrho\,\delta(\varrho)J_0(\omega\varrho)\omega\sin\omega r\,, \tag{5.1}$$

where $\varrho = b/k$.

Eq. (2.8) may serve to check the consistency of the procedure by evaluating the successive order of the expansion.

Some examples of solutions of Eq. (5.1) are given in the following table:

$-2k\delta(\varrho)$	$K_0(a\varrho)$	$\frac{\sqrt{\pi}}{2a}\exp[-a^2\varrho^2]$	$\frac{\exp[-a\varrho]}{\varrho}$	$\exp[-a\varrho]$	$\begin{cases} A & \varrho \leqslant R \\ 0 & \varrho > R \end{cases}$	
$U(r)$	$\frac{\exp[-ar]}{r}$	$\exp[-a^2r^2]$	$\frac{2a}{\pi}\frac{K_1(ar)}{r}$	$\frac{2a}{\pi}K_0(ar)$	$-\frac{2A}{\pi}\frac{1}{(R^2-r^2)^{\frac{1}{2}}}$	$r < R$
					0	$r > R$

The expansions obtained above can be extended to all other cases in which a Gel'fand and Levitan equation is known to exist. This applies for instance for collisions involving a tensor force ([6]).

([6]) R. G. Newton: *Phys. Rev.*, **100**, 412 (1955).

Similar expansions for the Dirac equations will be treated in a successive paper. For many-body collisions it is probably possible to use analogous methods, although the presence of many degrees of freedom must lead to a very complicated spectral function and the moments which appear in the expansions will be linked together by more intricated relations than those obtained here for one degree of freedom.

* * *

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RIASSUNTO (*)

Si derivano alcuni sviluppi asintotici degli spostamenti di fase corrispondenti ad energie elevate ed uno scattering di potenziale non relativistico. Sono validi sia per elevati che per bassi valori dei momenti angolari e si ottengono per mezzo dell'operatore di Gel'fand e Levitan che dà una connessione uniforme di due basi complete nello spazio hilbertiano. Si presenta anche un'approssimazione dell'ampiezza totale di scattering la cui validità si estende a tutti gli angoli di scattering ad eccezione delle vicinanze di 180°. Si fanno alcune osservazioni sul problema di ottenere il potenziale dalla conoscenza degli spostamenti di fase per una data energia in funzione del parametro d'urto.

(*) *Traduzione a cura della Redazione.*

NOTE TECNICHE

Osservazioni sul funzionamento di una Camera di Wilson con surcompressione.

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Riassunto. — Viene descritto un tipo particolarmente semplice di camera di Wilson surcompressa, ottenuta modificando una camera di Wilson normale. Si studiano i fenomeni connessi con la surcompressione e si analizzano i parametri da cui dipende la riduzione del tempo morto e l'efficacia di chiarificazione della camera. Vengono infine stabilite le condizioni da rispettare per la realizzazione di un ciclo di surcompressione.

1. – Introduzione.

La riduzione del tempo morto di una camera di Wilson per mezzo della surcompressione fu realizzata per la prima volta da Gaertner e Yeater [1]: il metodo da essi adottato consiste sostanzialmente nel far seguire all'espansione rapida una ricompressione che porta per alcuni decimi di secondo la pressione del gas della camera ad un valore superiore a quello preesistente all'atto dell'espansione; le condizioni iniziali di temperatura e di pressione vengono successivamente ristabilite mediante il ritorno lento del pistone alla posizione di compressione normale.

In vista dell'evidente interesse offerto dalla riduzione del tempo morto della camera di Wilson impiegata in esperienze con macchine acceleratrici, ed in ricerche con raggi cosmici quando gli eventi da studiare abbiano fre-

(*) Dal 1° Settembre 1956 presso l'Istituto di Fisica dell'Università di Indiana.
(1) E. R. Gaertner e M. L. Yeater: *Rev. Scient. Instr.*, **20**, 588 (1949).

quenze molto elevate, altri sperimentatori [2-7] hanno realizzato camere basate sullo stesso principio, ed alcuni di essi [1-9] ne hanno studiato sistematicamente il funzionamento.

I risultati fino ad oggi conseguiti, benchè ancora scarsi, sembrano mettere in evidenza che la camera surcompressa possiede un'estrema stabilità di funzionamento e comunque non richiede maggiori controlli di una normale camera di Wilson.

Con lo scopo di renderci conto delle specifiche difficoltà della surcompressione e per esaminare la possibilità di applicarla ad una grande camera, abbiamo realizzato una piccola camera sperimentale [10] la cui struttura ed il cui comando permettessero una grande flessibilità di funzionamento.

Nel presente lavoro, dopo aver dato una sommaria descrizione della struttura della camera e delle principali caratteristiche che la differenziano dalle altre già esistenti, vengono riferite alcune osservazioni generali sul funzionamento delle camere di Wilson con surcompressione.

2. – Descrizione della camera.

La camera di Wilson impiegata, è stata ottenuta modificandone una [11] a pistone di tipo normale, profonda 6 cm e dal diametro interno di 25 cm.

La Fig. 1 mostra la sezione assiale verticale della camera: *a*) è il pistone, *b*) il diaframma forato, ricoperto verso l'interno della camera di velluto nero di seta, *c*) una membrana di gomma che separa il retrocamera dalla camera, *d*) un pneumatico che, gonfiandosi, fa avanzare il pistone fino a che questo non venga trattenuto dalla ghiera *e*) montata sul suo gambo; il cilindro *f*) avvitato più o meno sul fondo della camera permette di variare la distanza del pistone dal diaframma forato.

La valvola V_1 di espansione, permette lo svuotamento rapido del pneumatico *d*); la valvola V_2 di surcompressione permette di immettere, più o meno rapidamente, aria compressa nel retrocamera; la valvola V_3 di espansione lenta, collega con l'esterno il retrocamera e permette il ritorno della

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([2]) E. H. Burhop: *Nature* (*London*), **175**, 832 (1955).

([3]) G. Fornaca, P. Franzini e G. Rinaldi: *Suppl. Nuovo Cimento*, **4**, 969 (1956).

( ) N. C. Barford: *Conference on Recent Developments in Cloud Chamber and Associated Techniques*, University College, London (1955), p. 137.

([5]) J. R. Atkinson: *Conference on Recent Developments in Cloud Chamber and Associated Techniques*, University College, London (1955), p. 150.

([6]) E. L. Goldwasser e V. O. Nicolai: *Conference on Recent Developments in Cloud Chamber and Associated Techniques*, University College, London (1955), p. 152.

([7]) J. Walker, F. Inst, G. Tagliaferri, F. Inst e D. W. Hadley: *Journ. Sci. Instr.*, **33**, 113 (1956).

([8]) N. C. Barford: *CERN Symposium*, 1956, p. 35.

([9]) J. Walker, J. C. Bower e D. W. Hadley: *CERN Symposium*, 1956, p. 40.

([10]) S. de Petris, R. Giacconi, A. E. Sichirollo e C. Succi: *Suppl. Nuovo Cimento*, **4**, 977 (1956).

([11]) G. Polvani: *Ric. Scient.*, anno 12°, **1**, 410 (1941).

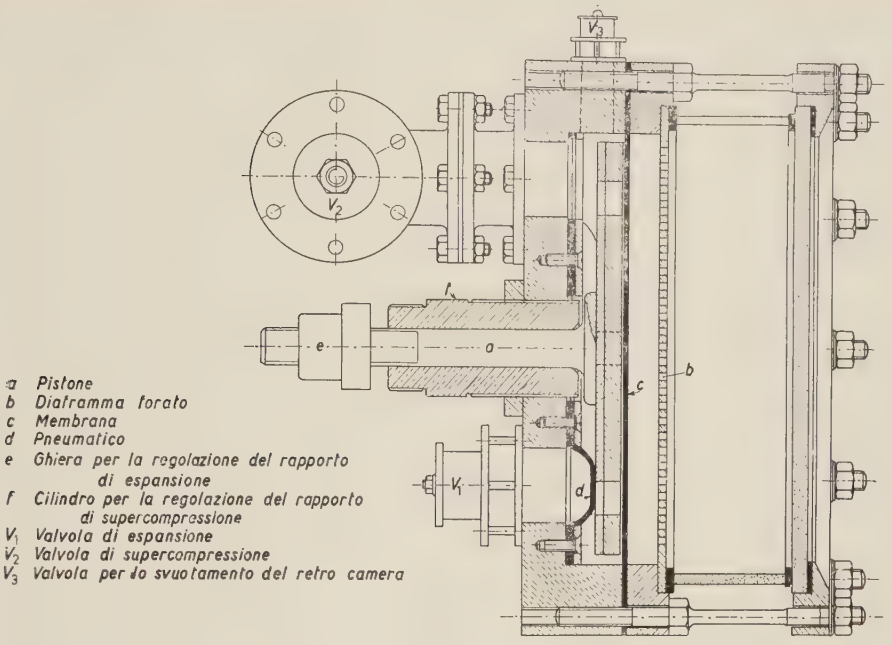

Fig. 1.

membrana dalla posizione di surcompressione a quella di compressione normale in un intervallo di tempo regolabile da 50 ms a qualche secondo.

La Fig. 2 mostra lo schema del sistema pneumatico d'alimentazione: C è

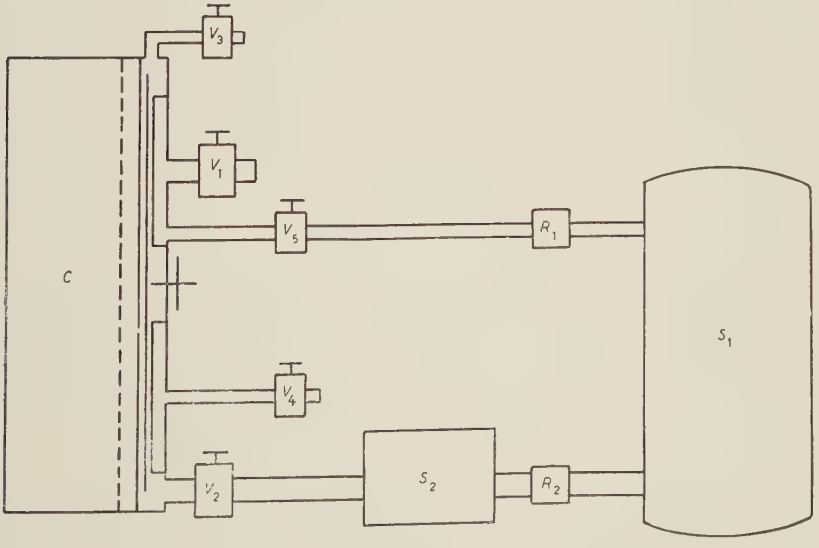

Fig. 2.

la camera; V_1, V_2 e V_3 sono rispettivamente le valvole di espansione, di surcompressione e di svuotamento del retrocamera; V_4 è la valvola d'espansione lenta per il funzionamento senza surcompressione e V_5 è la valvola che blocca l'afflusso dell'aria al pneumatico durante l'apertura di V_4. R_1 ed R_2 sono due riduttori di pressione: il primo fornisce aria ad una pressione di circa 3 atm al pneumatico P che provvede alla compressione normale, il secondo alimenta, ad una pressione variabile da 1.3 a 2 atm, il serbatoio S_2 da 10 dm³ che costituisce la riserva d'aria per la surcompressione; il serbatoio S_1 da 50 dm³, mantenuto ad una pressione compresa tra 5 e 7 atm, provvede all'alimentazione di tutto il sistema.

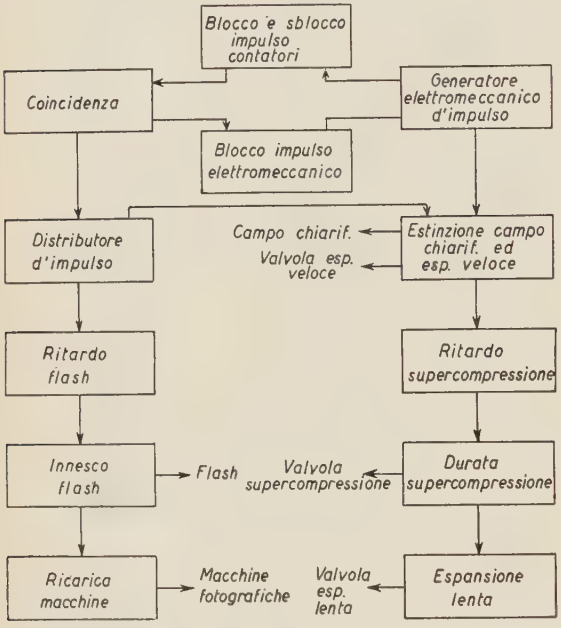

Fig. 3.

La Fig. 3, mostra lo schema a blocchi del comando che permette il funzionamento ciclico della camera sia mediante impulso fornito dalla camma di un motorino, di velocità regolabile da 2 a 10 giri al minuto primo, sia mediante impulso proveniente da una coincidenza di contatori; in questo secondo caso un dispositivo elettronico-meccanico di blocco e sblocco permette all'impulso proveniente dalla coincidenza di far espandere la camera solo durante un intervallo di pochi decimi di secondo tra la fine di un ciclo e l'inizio del successivo.

Quando l'impulso dei contatori trova via libera raggiunge il distributore e per mezzo di questo provvede sia alle operazioni di espansione e surcompressione, sia alle operazioni fotografiche. Quando invece, durante l'intervallo prestabilito, non si ha nessun impulso di coincidenza, il motorino provvede ad eccitare la sola catena delle operazioni di espansione e surcompressione.

3. – Funzionamento della camera.

La camera può venir fatta funzionare con o senza surcompressione; in ogni caso l'inizio e la durata delle varie operazioni possono venire variate entro limiti notevoli.

3·1. *Funzionamento come camera normale.* – La camera ha un tempo morto di circa 60 s e richiede una espansione lenta che normalmente viene eseguita 20 s dopo l'espansione rapida.

In condizioni di camera espansa, la membrana è mantenuta aderente al

pistone dalla pressione interna: la compressione si ottiene immettendo (Fig. 2) aria attraverso il riduttore R_1 nel pneumatico che, gonfiandosi, fa avanzare il pistone fino alla posizione determinata dalla ghiera e) posta sul suo gambo; l'espansione rapida si ottiene aprendo la valvola V_1 che scarica l'aria del pneumatico; l'espansione lenta si ottiene mediante la valvola V_4, comandata direttamente dalle camme del motorino che pilota il ciclo.

3·2. *Funzionamento come camera surcompressa.* – La camera è stata impiegata sia in posizione verticale che in posizione orizzontale: la surcompressione ha permesso di ridurre il tempo morto rispettivamente a 17 s ed a 7 s.

Immediatamente dopo l'espansione la membrana si trova adagiata sul pistone che è in posizione arretrata; per ottenere la surcompressione il retrocamera viene messo in comunicazione con il serbatoio S_1 per mezzo della valvola V_2: la membrana viene allora proiettata contro il diaframma forato in un intervallo di tempo regolabile da circa 15 ms a mezzo secondo, ed ivi mantenuta fino all'istante in cui viene aperta la valvola V_3. Questa, mettendo il retrocamera in comunicazione con l'esterno, permette alla membrana di ritornare a contatto del pistone che nel frattempo è stato riportato dal pneumatico in posizione di compressione normale: la durata di questa operazione, che realizza l'espansione lenta, può venir variata regolando l'apertura della valvola V_3.

La camera era riempita con argon saturato con una miscela di alcool ed acqua nel rapporto 3 : 1 alla pressione di 84 cm di Hg; le migliori condizioni di funzionamento si sono ottenute con rapporto di surcompressione uguale al rapporto di espansione, distribuendo nel tempo le operazioni come mostrato in Tabella I.

Tabella I.

	Camera verticale	Camera orizzontale	
Inizio espansione veloce . . .	0 s	0 s	Durata espansione veloce
Fine espansione veloce	0.012 s	0.012 s	
Flash	0.12 s	0.12 s	Permanenza in espansione
Inizio surcompressione	1.5 s	0.3 s	Durata surcompressione
Fine surcompressione	1.8 s	0.6 s	Permanenza in surcompressione
Inizio espansione lenta	3.3 s	1 s	Durata espansione lenta
Fine espansione lenta	4.5 s	2 s	
Inizio espansione veloce successiva	17 s	7 s	Durata della caduta delle gocce

4. – Esperienze ed osservazioni.

L'esperienza ha mostrato che per liberare il gas della camera di Wilson dai nuclei di condensazione che si formano dopo un'espansione si può ricorrere ad una serie di espansioni lente od alla surcompressione.

Come indice dell'efficacia del processo di chiarificazione, si è assunto il rapporto tra il valor medio della densità di gocce della nebbia di fondo che si osserva nell'espansione che segue le operazioni di chiarificazione ed il corrispondente valore in espansioni molto dilazionate tra di loro ed intervallate da almeno tre espansioni lente. Tale indice veniva rilevato fotografando una zona di camera di 1.5 cm di profondità, illuminata intensamente; ma in molti casi era sufficiente la semplice stima visuale.

BARFORD [4,5], ATKINSON [5], GOLDWASSER e NICOLAI [6] e WALKER *et al.* [7,9] hanno avanzato alcune ipotesi circa il modo con cui la surcompressione permette di ridurre il tempo morto della camera di Wilson. Dalle osservazioni dei precedenti autori si può concludere che i parametri più importanti da cui dipende, per una data camera, l'efficacia e la durata del ciclo di surcompressione sono:

1) stato del gas della camera alla fine del ciclo;

2) intensità della radiazione ionizzante che investe la camera;

3) permanenza della camera in espansione;

4) velocità della surcompressione;

5) rapporto di surcompressione;

6) permanenza della camera in surcompressione;

7) durata dell'espansione lenta;

8) intensità del campo chiarificatore;

9) intervallo di tempo necessario per la completa caduta delle gocce;

10) intervallo di tempo che intercorre tra la fine della caduta delle gocce ed il ristabilimento delle condizioni termiche iniziali.

Nelle prossime osservazioni i parametri sopra elencati verranno esaminati nell'ordine suggerito dalle prove eseguite e dalle osservazioni tratte da esse.

Nel ciclo ideale di una camera surcompressa, lo stato del gas alla fine della deposizione delle gocce prodottesi nell'espansione lenta dovrebbe identificarsi con lo stato iniziale; in altre parole, l'ultimo dei parametri sopra elencati dovrebbe essere nullo.

Poichè tra tutte le operazioni del ciclo la deposizione delle gocce è quella che occupa la maggior parte del tempo, se ne è voluta misurare accuratamente la durata: l'esperienza ha dimostrato che essa può venire sensibilmente ridotta eseguendo l'espansione lenta piuttosto « rapidamente »; nel nostro caso in circa 1.2 s per camera verticale ed in circa 1 s per camera orizzontale.

Questo fatto potrebbe venir spiegato osservando che i nuclei di rievaporazione [12] hanno, anche dopo una surcompressione, purchè non eccessivamente protratta nel tempo, dimensioni notevolmente superiori a quelle dei nuclei di condensazione su cui si originano le tracce; l'aumento della rapidità dell'espansione lenta permetterebbe quindi di raggiungere valori del rap-

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[12] J. C. WILSON: *The Principles of Cloud Chamber Techniques* (Cambridge, 1951), page 13.

porto S/S_n (S sovrassaturazione raggiunta nell'espansione lenta, S_n sovrassaturazione minima necessaria per l'ingrossamento dei nuclei di rievaporazione) molto elevati e così un notevole aumento della velocità di ingrossamento delle gocce, e conseguentemente della velocità e dell'uniformità di caduta.

Osservando che in un ciclo ideale il gas della camera scambia calore con l'esterno praticamente solo durante la permanenza in espansione e in surcompressione e durante l'espansione lenta, se ne deduce che, per poter chiudere completamente il ciclo cioè per riportare la camera nelle condizioni iniziali di funzionamento, occorre che la quantità di calore assorbita durante la permanenza in espansione sia uguale alla quantità di calore ceduta durante la permanenza in surcompressione e durante l'espansione lenta. Poichè la permanenza in espansione non può essere, per ragioni fotografiche, inferiore ad un decimo di secondo, ne segue che, per un determinato rapporto di surcompressione e per una espansione lenta di velocità fissata, la durata della permanenza in surcompressione non può essere inferiore ad un certo valore. I risultati sperimentali di WALKER *et al.* [7] ed i nostri, mettono in evidenza che ordinariamente si usano, per camere funzionanti a ciclo, rapporto e durata della surcompressione più elevati di quelli richiesti dalla permanenza in espansione, raggiungendo così dei bilanci di calore negativi.

Per studiare specificamente l'effetto della surcompressione si è variato separatamente il valore del rapporto di surcompressione e la permanenza in essa; la camera è stata impiegata come camera di Wilson normale, con un ciclo di due minuti di durata e tre espansioni lente; ogni 15 o 20 espansioni si faceva seguire all'espansione rapida una surcompressione ed immediatamente dopo, un'espansione, lenta molto rapida ($\simeq 1/10$ di s); con questo metodo si è sorpresa, con sufficiente evidenza, la persistenza di tracce già osservate nell'espansione rapida precedente, anche con valori piuttosto elevati del rapporto di surcompressione.

D'altra parte se, dopo l'espansione rapida, si surcomprime per qualche secondo, la successiva espansione lenta mostra una condensazione sensibilmente indipendente dalla durata e dal valore del rapporto di surcompressione.

I presenti risultati, oltre che confermare quelli già ottenuti da WALKER *et al.* [7-9] e da HAZEN [4], uniti all'ipotesi, suggerita da THOMPSON [12], che le gocce in evaporazione raggiungano, anche alla fine della surcompressione, raggi di equilibrio sensibilmente maggiori di quelli dei nuclei ionici di condensazione, fanno ritenere che la surcompressione è da sola inefficace all'eliminazione dei germi residui di gocce in evaporazione.

É stato suggerito [1,4] che il campo chiarificatore, asportando i nuclei di rievaporazione carichi, abbia una grande efficacia nel funzionamento della camera surcompressa; per saggiare tale ipotesi abbiamo esaminato l'azione chiarificatrice del campo, variandone l'intensità, in presenza di un forte carico ionico, da 20 V/cm a 100 V/cm.

I risultati conseguiti sembrano suggerire che l'ipotesi proposta sia solo parzialmente valida, e ciò probabilmente è in connessione con il fatto che molte gocce possono perdere la loro carica e che inoltre molti nuclei di rievaporazione hanno ancora dimensioni tanto grandi da non potersi muovere con sufficiente rapidità sotto l'azione del campo.

La velocità di surcompressione è stata variata entro vasti limiti osservando che velocità troppo elevate, dell'ordine della velocità dell'espansione rapida, producevano un fondo che, dopo serie di due o tre espansioni, diveniva intol-

lerabile. Ciò potrebbe venir spiegato ammettendo ([13,14]) che il troppo brusco passaggio del gas attraverso i fori del diaframma e la trama del velluto, creando elettrizzazioni e locali gradienti di temperatura, possa portare alla formazione di numerosi germi di condensazione.

Si è infine osservato che, se si aumentava di molto, a ciclo regolato, l'intensità della radiazione ionizzante incidente, la nebbia di fondo cresceva diventando intollerabile in 5-10 espansioni. Il fenomeno è probabilmente connesso al fatto che, quando la densità dei nuclei di condensazione carichi diventa troppo elevata, la permanenza in surcompressione non è sufficientemente lunga per far evaporare la quantità di miscela necessaria alla loro totale deposizione durante l'espansione lenta. Con questa ipotesi è abbastanza ovvio interpretare il fatto sperimentale che con carichi ionici molto intensi si debbano aumentare solo le durate di permanenza in espansione ed in surcompressione, senza variare la velocità dell'espansione lenta.

5. – Conclusioni.

Dalle osservazioni ora esposte si possono trarre le seguenti conclusioni: la surcompressione riduce il tempo morto di una camera di Wilson essenzialmente per due ragioni: 1) perchè permette di realizzare rapidamente un ciclo termodinamicamente chiuso; 2) perchè l'espansione lenta si effettua in condizioni tali da lasciare il gas della camera saturo, eliminando l'intervallo di tempo necessario, in una camera di Wilson normale, alla ridiffusione del vapore condensatosi durante le espansioni lente.

Per quanto riguarda la realizzazione pratica di un ciclo di surcompressione le sole condizioni da rispettare sono le seguenti:

a) la permanenza della camera in espansione deve venire regolata in base al rapporto ed alla permanenza in surcompressione ed alla durata dell'espansione lenta;

b) la permanenza in surcompressione, specialmente in presenza di carichi ionici elevati, deve venir prolungata per rendere disponibile una quantità di vapore sufficiente all'ingrossamento di tutti i germi residui di gocce;

c) l'espansione lenta deve essere abbastanza rapida per assicurare una sovrassaturazione sufficiente a garantire la condensazione e l'ingrossamento di tutti i nuclei di rievaporazione;

d) la velocità con cui si effettua la surcompressione non deve superare un certo limite connesso essenzialmente alla struttura della camera.

Da quanto precede risulta che la surcompressione può venir applicata unicamente a camere di Wilson che già funzionino soddisfacentemente con una sola espansione lenta. Poichè d'altra parte il funzionamento di una camera surcompressa non è più critico di quello di una camera normale, la surcom-

([13]) C. R. EMIGH: *Rev. Sci. Instr.*, **25**, 221 (1954).

([14]) C. R. EMIGH e P. C. FISHER: *Conference on Recent Developments in Cloud Chamber and Associated Techniques*, University College, London (1955), p. 155.

pressione è applicabile ad ogni camera di Wilson, anche di grandi dimensioni, tanto più che è relativamente facile ideare sistemi semplici che permettano di soddisfarne le limitate esigenze di rapidità e prontezza.

* * *

Ci è gradito dovere ringraziare i Professori G. Polvani e P. Caldirola per il loro interessamento a questa ricerca.

L'esperienza è stata realizzata oltre che con gli aiuti finanziari dell'Istituto di Scienze Fisiche e della Sezione di Milano dell'I.N.F.N., anche con quelli degli Amici dell'Istituto e dell'A.N.I.D.E.L.

SUMMARY

An overcompressed Wilson cloud chamber of a particularly simple type obtained by modifying a conventional Wilson cloud chamber is described. Phenomena connected with the overcompression have been studied and the parameters from which is deduced the recycling time reduction and the clearing efficiency of the chamber analysed. We have determined the conditions necessary for the realization of an overcompression cycle.

LETTERE ALLA REDAZIONE

(La responsabilità scientifica degli scritti inseriti in questa rubrica è completamente lasciata dalla Direzione del periodico ai singoli autori)

The Model of the Electron.

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(ricevuto il 5 Aprile 1957)

The proposed model of the electron consists of a magnetic disk rolling without sliding having radius « r », uniformly distributed mass « m » and a « point » charge « e » on the periphery of the disk. Such a model enables the following conclusions to be drawn:

1. – *a*) The electron charge moves along the cycloid and the length $2\pi r$ characteristic for the cycloid may be treated as de Broglie's wave length

$$\lambda = 2\pi r = \frac{h}{mV} . \tag{1.1}$$

b) The velocity « V » of the electron is equal to the rotational velocity « v » (spin velocity) of the electric charge against the centre of mass of the electron.

$$V = v . \tag{1.2}$$

c) From (1.1) and (1.2) it results that the angular momentum (spin) is constant and equal

$$\frac{1}{2}\frac{h}{2\pi} . \tag{1.3}$$

d) The magnetic moment

$$M = r^2 e/cT = eh/4\pi mc ,$$

as $v = 2\pi r/T$.

e) The constancy of the angular momentum and the equality $v = V$ lead to the conclusion that the dimension (r) of the electron is variable depending on the velocity of the electron.

2. – The magnetic disk has its own magnetic field. At the point A (Fig. 1) the magnetic field strength is

$$H = M/r^3 = ve/2cr^2 .$$

The electric charge at A having the relative velocity $-V$ against the resting magnetic field undergoes the magnetoelectric force F. The magnetoelectric field intensity

$$E_m = F/e = ev^2/2r^2c^2 ,$$

is proportional to « a » (Fig. 1). The equation of « a » has the form of the Schrödinger wave equation for the free particle. Hence it can be concluded that the wave function ψ is proportional to the internal magnetoelectric field strength of the electron.

3. – It is known that the electron's ability to interact with its surroundings has a wave and corpuscular character. The ability to act in this double character must exist in the electron independently of the method in which the experiment is carried out. From the point of view

of the proposed model it is interesting to establish which phase of the charge rotation may correspond to the corpuscular and which one to the wave action of the electron with the surroundings.

It may be easily established from the model that:

$$E = h \cdot \nu = mv^2 . \tag{5.1}$$

This result differs from the well known

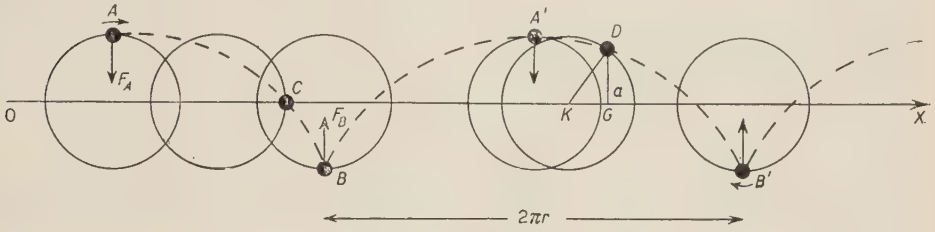

Fig. 1.

Considering the electron energy in the phase, when the charge is in the point C, we can assume that as the strength of the magnetoelectric field is zero, the total energy allowed for exchange with the surroundings has the form of kinetic (mechanical) energy. Such a state we will denote the mechanical phase of the electron. The state when ψ = maximum we shall call the field phase of the electron. In this way we receive automatically the answer to the question which phase of rotation of the charge corresponds to the mechanical or to the wave character of the electron's interaction with its surroundings. In other words from our point of view inside the electron a permanent conversion of mechanical energy into field energy and conversely takes place.

4. – We introduce an additional assumption. The electron is able to exchange its energy with its surroundings only in those moments, when it possesses mechanical or field phase. The indeterminacy principle is a consequence of this assumption.

5. – When considering from the wave view point the problem of the mutual action between an electron with energy E and its surroundings we ascribe to the electron the energy $E = h \cdot \nu$. formula

$$h \cdot \nu = \tfrac{1}{2} m' v^2 . \tag{5.2}$$

The measurements of the velocity of the electron, which belong to the corpuscular type of measurements verify the equation (5.2). But we must remember that the equation (5.2) includes the information about two different kinds of acts, which take place. Due to the law of energy conservation the sign of equality in this equation represents only the bridge between the wave photon and the corpuscular action of the electron with the surroundings.

From the mechanical point of view based on the model the energy of the moving electron must be:

$$E = E_{\text{kin.}} + E_{\text{kin.rot.}} + E_{\text{pot.}} . \tag{5.3}$$

Where $E_{\text{kin.}}$ is the translation energy, $E_{\text{kin.rot.}}$ — the energy of rotation and $E_{\text{pot.}}$ — the potential energy connected with the changes of dimensions of the magnetic disk.

On the other hand considering the mechanical (corpuscular) action we ascribe to the electron the energy

$$E = \tfrac{1}{2} m' v^2 .$$

The equations (5.1) and (5.3) express the

same value of the energy. If it is so, a new suggestion automatically arises, namely that the sum of

$$E_{\text{kin. rot.}} + E_{\text{pot.}} = \tfrac{1}{2} \cdot mv^2 ,$$

characterizes the energy of internal electron transformation leading to an increase in the mass of the electron

$$\Delta m = mv^2/2c^2 .$$

Then for small velocities it can be easily established that:

$$m' = \frac{m}{\sqrt{1 - v^2/c^2}} .$$

6. – The magnetoelectric field strength in the field phase may be combined with an auxiliary electric dipole with a moment « L » placed perpendicularly to the « $\overline{V}$ ». Then at the point A (Fig. 1) the electric dipole gives an electric field with the intensity

$$\frac{2L}{r^3} = \frac{1}{2} \frac{ev^2}{r^2 c^2} . \tag{6.1}$$

Inserting $r = h/2\pi mv$ we obtain $L = \tfrac{1}{2} \cdot (v/c) M$. With regard to the directions of acting forces we can write:

$$\boldsymbol{L} = \frac{1}{2} \left(\frac{\boldsymbol{v}}{c} \times \boldsymbol{M} \right) . \tag{6.2}$$

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On the Question of Possible Charge Properties of Weak Interactions (*).

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(ricevuto il 29 Aprile 1957)

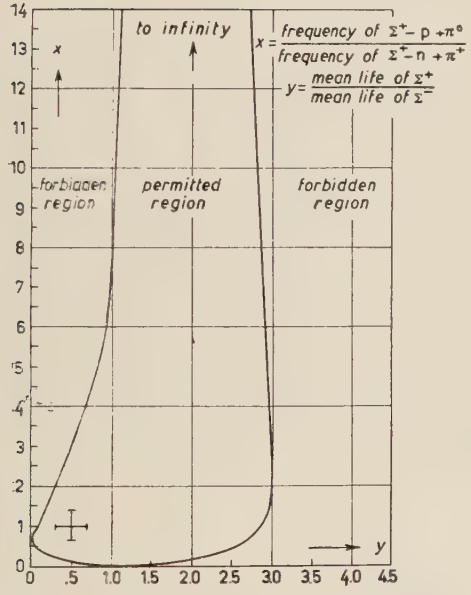

Fig. 1. – The permitted region in the X-Y plane.

It has been shown [1] that, if parity and time reversal are conserved in the decay of the Σ particles, the observed lifetime ratio $Y =$ (mean life of Σ^+)/(mean life of Σ^-) and the observed branching ratio $X =$ (frequency of $\Sigma^+ \to \to p + \pi^0$)/(frequency of $\Sigma^+ \to n + \pi^-$) cannot be accounted for with a decay interaction transforming as a component of a spherical tensor of rank $\frac{1}{2}$ ($\Delta T = \frac{1}{2}$ rule) [2-6]. Present experimental evidence of parity non-conservation in the β decay [7], in the $\pi \to \mu + \nu$ decay and in the $\mu \to e + \nu + \nu$ decay [8], and the less direct evidence from the τ and θ decays [9], encourages the hypothesis that also in the weak decay interactions of hyperons and K-mesons parity is not conserved. It was shown that, if the decay interaction satisfies $\Delta T = \frac{1}{2}$ and no other assumptions are made, the point $P \equiv (Y, X)$ in

(*) This work was performed under the auspices of the U. S. Atomic Energy Commission.

(+) On leave of absence from Istituto di Fisica dell'Università di Roma, Italy.

(1) W. Alvarez, H. Bradner, P. Falk-Vairant, J. D. Gow, A. H. Rosenfeld, F. T. Solmitz and R. D. Tripp: *Nuovo Cimento* (in press).

(2) R. Gatto: *Nuovo Cimento*, **3**, 318 (1956).

(3) G. Wentzel: *Phys. Rev.*, **101**, 1215 (1956).

(4) M. Gell-Mann: *Proceedings of the Rochester Conference* (1956), Sect. **8**, p. 25.

(5) R. H. Dalitz: *Proc. Phys. Soc.*, 527 (1956).

(6) T. Kotani: to be published.

(7) C. S. Wu, O. E. Ambler, R. P. Hudson, R. W. Hayward and D. D. Hoppes: *Phys. Rev.* (in press.).

(8) R. L. Garwin, L. M. Lederman and M. Weinrich: *Phys. Rev.* (in press).

(9) R. H. Dalitz: *Proceedings of the Rochester Conference* (1956), Sect. **8**, p. 19.

the X-Y plane is limited to a « permitted region » of the plane ([2]), as reported in Fig. 1. It is seen from Fig. 1 that the experimental point lies inside the permitted region. It is well known that, if time reversal is satisfied, the ensuing symmetry condition on the S matrix limits the form of the decay matrix element by a theorem first used by WATSON in the interpretation of photomeson production ([10]). This limitation, in the case of parity conservation, strongly restricts the « permitted region » in the Y-X plane, which is reduced to a line, depending on the assumed spin and parity of the Σ. We shall show that in the case under discussion of parity non-conservation the « permitted region » in Fig. 1 is not essentially reduced by the further requirement of time-reversal invariance.

Consider first the decay of a Σ with spin $\frac{1}{2}$. The possible states for the final pion-nucleon system are $s_{\frac{1}{2}}$ with $T = \frac{3}{2}$ and $\frac{1}{2}$, and $p_{\frac{1}{2}}$ with $T = \frac{3}{2}$ and $\frac{1}{2}$. If time reversal is satisfied the $\Delta T = \frac{1}{2}$ rule leads to the following expressions for the total decay probabilities of the Σ particles:

$$(1) \qquad W(\Sigma^+|\mathrm{p}0) = \frac{2}{3}(A_3^2 + A_{31}^2) + \frac{1}{3}(A_1^2 + A_{11}^2) - \frac{2\sqrt{2}}{3}\left[A_3 A_1 \cos(\alpha_3 - \alpha_1) + A_{31}A_{11}\cos(\alpha_{31} - \alpha_{11})\right],$$

$$(1') \qquad W(\Sigma^+|\mathrm{n}+) = \frac{1}{3}(A_3^2 + A_{31}^2) + \frac{2}{3}(A_1^2 + A_{11}^2) + \frac{2\sqrt{2}}{3}\left[A_3 A_1 \cos(\alpha_3 - \alpha_1) + A_{31}A_{11}\cos(\alpha_{31} - \alpha_{11})\right],$$

$$(1'') \qquad W(\Sigma^-|\mathrm{n}-) = 3(A_3^2 + A_{31}^2).$$

The quantities A are real numbers and the α are the appropriate pion-nucleon phase shifts at the Σ decay energy. The indices refer to the final pion-nucleon states according to the usual convention. From Eqs. (1) we can write for the measurable quantities Y and X

$$(2) \qquad Y = \frac{3}{1 + \xi^2},$$

$$(2') \qquad X = \frac{2 + \xi^2 - 2\sqrt{2}\xi\eta}{1 + 2\xi^2 + 2\sqrt{2}\xi\eta},$$

where

$$(3) \qquad \xi = \left(\frac{A_1^2 + A_{11}^2}{A_3^2 + A_{31}^2}\right)^{\frac{1}{2}},$$

and

$$(3') \qquad \eta = \frac{A_3 A_1 \cos(\alpha_3 - \alpha_1) + A_{31}A_{11}\cos(\alpha_{31} - \alpha_{11})}{\sqrt{A_3^2 + A_{31}^2}\sqrt{A_1^2 + A_{11}^2}}.$$

([10]) K. M. WATSON: *Phys. Rev.*, **95**, 228 (1954).

From Eq. (3′) it can be shown that for any value of the real quantities A_1, A_3, A_{11} and A_{31} we have

$$\eta^2 \leqslant \max\left[\cos(\alpha_3 - \alpha_1), \cos(\alpha_{31} - \alpha_{11})\right],$$

where $\max[a, b]$ means the larger of the two values a and b. At the energy corresponding to the Σ decay energy, $\cos(\alpha_3 - \alpha_1) \cong 0.95$ and $\cos(\alpha_{31} - \alpha_{11}) \cong 0.99$. However, as evident from Eq. (2), η^2 is always less than unity. Therefore the permitted region of Fig. 1 is not essentially restricted. The same conclusion is found to hold also for the higher spin values. We therefore conclude that, if parity is not conserved in the Σ decays, the experimental values of X and Y are not in disagreement with the $\Delta T = \frac{1}{2}$ rule. However it must be remarked that the conditions imposed in such case from the $\Delta T = \frac{1}{2}$ rule are not very stringent, so that the non-disagreement does not provide any conclusive evidence.

The more stringent condition imposed by the $\Delta T = \frac{1}{2}$ rule on the Λ^0 branching ratio,

$$w(\Lambda^0 \to \mathrm{p}\pi^-) = 2w(\Lambda^0 \to \mathrm{n}\pi^0),$$

seems to be satisfied by the recent data of STEINBERGER, who finds a value of $(66 \pm 5)\%$ for the fraction of Λ^0 decaying into $\mathrm{p} + \pi^-$ ([11]). Moreover, the latest data ([12]) for the $(\tau^+ \to \pi^+ + \pi^- + \pi^+)/(\tau^+ \to \pi^+ + \pi^0 + \pi^0)$ ratio, giving a value $\cong 0.39 \pm 0.09$, do not contradict the value predicted for such ratio for a spin-zero τ-meson from the $\Delta T = \frac{1}{2}$ rule, namely a value $\gtrsim \frac{1}{4}$. This conclusion, however, is not stringent since it also follows if a $\Delta T = \frac{3}{2}$ contribution is present. As is well known ([2,3]), the $\Delta T = \frac{1}{2}$ rule forbids the $\mathrm{K}^+ \to \pi^+ + \pi^0$ decay mode for K^+ of even spin in the absence of electromagnetic interactions. The transition probability for such process is known experimentally to be much lower than that of $\mathrm{K}^0 \to \pi^+ + \pi^-$. It has been pointed out by Gell-Mann that the electromagnetic corrections may be too small to account for the observed $\mathrm{K}^+ \to \pi^+ + \pi^0$ rate ([13]). These small $\Delta T = \frac{3}{2}$ and $\Delta T = \frac{5}{2}$ amplitudes are related to the observed ratios

$$f = w(\mathrm{K}_1^0 \to 2\pi^0)/w(\mathrm{K}_1^0 \to \pi^+ + \pi^-),$$

and

$$g = w(\mathrm{K}^+ \to \pi^+ + \pi^0)/w(\mathrm{K}_1^0 \to 2\pi),$$

by

$$f = \tfrac{1}{2}\left(1 + 3\sqrt{2}(x_3 - x_5)\cos(\alpha_2 - \alpha_0)\right), \tag{4}$$

$$g = \tfrac{3}{4}(x_3 + \tfrac{2}{3}x_5)^2. \tag{4′}$$

Here we denote by $x_3 \exp[i(\alpha_2 - \alpha_0)]$ the ratio of the reduced matrix element of the $\Delta T = \frac{3}{2}$ transition to the reduced matrix element of the $\Delta T = \frac{1}{2}$ transition, and by $x_5 \exp\, i(\alpha_2 - \alpha_0)]$ the similar ratio of $\Delta T = \frac{5}{2}$ to $\Delta T = \frac{1}{2}$. The value of g can

([11]) Private communication from A. ROSENFELD.

([12]) R. W. BIRGE, D. H. PERKINS, J. R. PETERSON, D. H. STORK and M. N. WHITEHEAD: *Nuovo Cimento*, **4**, 834 (1956).

([13]) M. GELL-MANN: preprint.

be determined from the data if we assume that only one K-meson exists. In this case g is found to be $\sim 1/(430 \pm 100)$ from the observed ratio of the K^0 and K^+ lifetimes ($\tau_{K^0} = (1.0^{+0.3}_{-0.2}) \cdot 10^{-10}$ s [14], $\tau_{K^+} = (1.24 \pm 0.02) \cdot 10^{-8}$ s [15]), and from the measured [12] $w(K^+ \to \text{other final states})/w(K^+ \to \pi^+ + \pi^0) = 2.5 \pm 0.4$. The most recent experimental value of f is $f \cong 1/16$, due to Steinberger. Assuming again the symmetry of the S-matrix, α_2 and α_0 are the phase shifts of the final pion-pion system in $l = 0$, $T = 2$ and in $l = 0$, $T = 0$ respectively. The final pion energies are near the resonance proposed to explain the second pion-proton maximum [16]. According to Dyson such resonance could occur in the $T = 0$ state. If α_0 is near 90° and α_2 small, so that we have $\cos(\alpha_2 - \alpha_0) \cong 0$, it is clearly impossible to explain the observed f ratio with small $\Delta T = \frac{3}{2}$ and $\Delta T = \frac{5}{2}$ contributions. If we tentatively assume that the two phase shifts are sufficiently small, so that we have $\cos(\alpha_2 - \alpha_0) \cong 1$, we find from Eq. (10) the two sets of solutions $x_3 \simeq -0.05$, $x_5 \cong 0.16$, and $x_3 \cong -0.12$, $x_5 \cong 0.09$. Such values may be too large to be accounted for as electromagnetic corrections (amplitudes of the order e^2 according to perturbation theory). Another difficulty, also mentioned by Gell-Mann, is that whereas the probability for $K^+ \to \pi^+ + \pi^0$ would be expected in this model to be proportional to e^4, that for $K^+ \to \pi^+ + \pi^0 + \gamma$ should turn out proportional to e^2. It must be remarked, however, that the phase space available to the $2\pi + \gamma$ final state is expected to be much smaller than that for 2π (the phase space for $2\pi + \gamma$ is about 5 times that for 3π). Moreover, for a zero spin K^+ the two final mesons are left in the $l = 1$, $P = -1$ state by an E1 (and also possibly by M1) γ-transition, and thus they have to overcome a centrifugal barrier. However, no cases at all of $K^+ \to 2\pi + \gamma$ have been reported so far.

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[14] *Rochester Conference* (1956), Sect. **6**, p. 30 and Sect. **5**, p. 5.
[15] W. ALVAREZ, CRAWFORD, GOOD and STEVENSON: private communications.
[16] F. J. DYSON: *Phys. Rev.*, **99**, 1037 (1955); G. TAKEDA: *Phys. Rev.*, **100**, 440 (1955).

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On the Generalized Ward Identity (*).

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Ward's identity (1) which shows the relation between the vertex operator with equal electron momenta and the electron propagator has been generalized for the case where the electron momenta are not equal. The generalized identity has not been rigorously proved, in spite of the fact that it is extensively used by many authors. The proof is given in this paper *without recourse to perturbation expansion or Feynman's diagram*. It is shown to be a consequence of the conservation of the current.

One can express Ward's identity in the following form, namely

$$\frac{1}{i}\frac{\partial S_0(p)}{\partial p^\mu} = -S_0(p)\Gamma_\mu(p;p)S_0(p)\,, \tag{1}$$

where the function $S_0(p)$ is the renormalized electron propagator and the $\Gamma_\mu(p;p)$ the renormalized vertex operator with equal electron momenta.

It has been suggested that equation (1) be generalized in the following manner (2).

$$S_0(p) - S_0(q) = -i(p-q)^\mu S_0(p)\Gamma_\mu(p;q)S_0(q)\,. \tag{2}$$

The generalized relation (2) has not been proved in a rigorous manner. The aim of this note is to prove the relation (2) by the use of the equations of motion for the electron and the photon.

According to the gauge invariance of the theory, the renormalized photon propagator

$$D_{\mu\nu}(x-x') \equiv \langle T(\boldsymbol{A}_\mu(x), \boldsymbol{A}_\nu(x'))\rangle_0 \ (**)\,, \tag{3}$$

(*) This work is supported by the National Science Foundation.

(1) J. C. Ward: *Phys. Rev.*, **77**, 293 (1950); **78**, 182 (1950); *Proc. Phys. Soc.*, **64**, 54 (1951).

(2) T. D. Lee: *Phys. Rev.*, **95**, 1329 (1954); H. S. Green: *Proc. Phys. Soc.*, **66**, 873 (1953); L. D. Landau and I. M. Khalatnikov: *J.E.T.P.*, **29** 89 (1955); *English Translation*, **2**, 69 (1956).

(**) Bold-faced letters will be used for the renormalized Heisenberg field operators, throughout this note. For the notation in this note, see Jauch and Rohrlich: *Theory of Photons and Electrons* (Cambridge, Mass., 1955).

satisfies

$$\partial^{\prime\prime} D_{\mu\nu}(x-x') = \partial_\nu D_c(x-x') \,, \tag{4}$$

where $D_c(x-x')$ is the Stückelberg-Feynman causal function defined as

$$D_c(x) = \frac{-i}{(2\pi)^4}\int \mathrm{d}^4k \exp\,[ikx]\,\frac{1}{k^2-i\varepsilon} \,. \tag{5}$$

The proof of the equation (4) is rather lengthy and will, therefore, be discussed later on.

The continuity equation of the current gives

$$\partial_y^\mu \langle T(\boldsymbol{J}_\mu(y), \boldsymbol{\psi}(x), \boldsymbol{\psi}(x'))\rangle_0 = \tag{6}$$
$$= e_0 \left\{ \langle T(\boldsymbol{\psi}(x), \boldsymbol{\psi}(y))\rangle_0 \,\delta(y-x') - \delta(x-y)\langle T(\boldsymbol{\psi}(y), \bar{\boldsymbol{\psi}}(x'))\rangle_0 \right\},$$

where e_0 is the renormalized charge and use has been made of the relation

$$\delta(x_0 - x_0')[\boldsymbol{\psi}(x), \boldsymbol{J}_0(x')] = e_0\,\delta(x-x')\boldsymbol{\psi}(x') \,. \tag{7}$$

If one defines the vertex operator $\Gamma_\mu(x-y;\, y-x')$ by

$$\langle T(\boldsymbol{\psi}(x), \bar{\boldsymbol{\psi}}(x'), \boldsymbol{A}_\mu(y))\rangle_0 \equiv \tag{8}$$
$$\equiv -e_0 \int \mathrm{d}^4\xi\, \mathrm{d}^4\eta\, \mathrm{d}^4\zeta S_0(x-\xi)\Gamma^\nu(\xi-\eta;\, \eta-\zeta)S_0(\zeta-x')D_{\nu\mu}(\eta-y) \,,$$

then, the equation (6) is written, due to (4), in terms of Γ_μ and S_0 as follows

$$e_0 \Box_y \partial_y^\mu \int \mathrm{d}^4\xi\, \mathrm{d}^4\eta\, \mathrm{d}^4\zeta S_0(x-\xi)\Gamma^\nu(\xi-\eta;\, \eta-\zeta)S_0(\zeta-x')D_{\nu\mu}(\eta-y) = \tag{9}$$
$$= i e_0 \int \mathrm{d}^4\xi\, \mathrm{d}^4\eta\, \mathrm{d}^4\zeta S_0(x-\xi)\Gamma_\nu(\xi-\eta;\, \eta-\zeta)S_0(\zeta-x')\,\partial_y^\nu \delta(\eta-y) =$$
$$= i e_0 \int \mathrm{d}^4\xi\, \mathrm{d}^4\eta\, \mathrm{d}^4\zeta S_0(x-\xi)\,\partial_y^\nu \Gamma_\nu(\xi-y;\, y-\zeta)S_0(\zeta-x') =$$
$$= e_0\{S_0(x-y)\,\delta(y-x') - \delta(x-y)S_0(y-x')\} \,.$$

Upon introducing the Fourier transform of the equation (9), one gets

$$-i(p-q)^\nu S_0(p)\Gamma_\nu(p;\, q)S_0(q) = S_0(p) - S_0(q) \,. \tag{10}$$

This proves the generalized Ward identity (*).

Let us return to the equation (4).

(*) The conjecture that Ward's identity would be a consequence of the gauge invariance of the theory was stated by Rohrlich. F. Rohrlich: *Phys. Rev.*, **80**, 666 (1950).

The total electromagnetic potential $\boldsymbol{A}_\mu(x)$ is split into two parts

$$\boldsymbol{A}_\mu(x) = \boldsymbol{a}_\mu(x) + \partial_\mu \Lambda(x) , \tag{11}$$

where

$$\begin{cases} \partial^\mu \boldsymbol{a}_\mu(x) = 0 , \\ (\Box \Lambda(x))^{(+)} \Phi = 0 . \end{cases} \tag{12}$$

From (12), the $\boldsymbol{a}_\mu(x)$ must satisfy the commutation relation

$$\langle [\boldsymbol{a}_\mu(x), \boldsymbol{a}_\nu(x')] \rangle_0 = -i \int_0^\infty \mathrm{d}a \varrho(a) \left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{a} \right) \Delta(x - x', a) ,$$

where $\varrho(a)$ is a spectral function introduced by Källén, Lehmann, and Gell-Mann and Low [3].

In a similar fashion, the $\Lambda(x)$ satisfies

$$\langle [\Lambda(x), \Lambda(x')] \rangle_0 = -i \int \mathrm{d}a \varrho_1(a) \Delta(x - x', a) . \tag{13}$$

The commutation relation of the total potential $\boldsymbol{A}_\mu(x)$ is

$$\langle [\boldsymbol{A}_\mu(x), \boldsymbol{A}_\nu(x')] \rangle_0 = \langle [\boldsymbol{a}_\mu(x), \boldsymbol{a}_\nu(x')] \rangle_0 + \partial_\mu \partial'_\nu \langle [\Lambda(x), \Lambda(x')] \rangle_0 = \tag{14}$$

$$= -i \int_0^\infty \mathrm{d}a \varrho(a) \left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{a} \right) \Delta(x - x', a) + i \int_0^\infty \mathrm{d}a \varrho_1(a) \partial_\mu \partial_\nu \Delta(x - x', a) .$$

If one compares (14) with the canonical commutation relation at $t = t'$ (*), the following relation will be obtained:

$$\begin{cases} M \equiv \int_0^\infty \mathrm{d}a \varrho(a)/a = -\int_0^\infty \mathrm{d}a \varrho_1(a) , \\ Z_3^{-1} = \int_0^\infty \mathrm{d}a \varrho(a) , \\ -1 = \int_0^\infty \mathrm{d}a \cdot a \cdot \varrho_1(a) . \end{cases} \tag{15}$$

[3] G. Källén: *Helv. Phys. Acta*, **25**, 417 (1952); H. Lehmann: *Nuovo Cimento*, **11**, 342 (1954); M. Gell-Mann and F. E. Low: *Phys. Rev.*, **95**, 1300 (1954).

(*) For instance, see Källén's article [3].

Consequently,

$$\text{(16)} \qquad \begin{cases} \varrho_1(a) = -\left(\dfrac{1}{a} + N\right) \delta(a) \,, \\ N = M - \displaystyle\int_0^\infty \mathrm{d}a \, \delta(a)/a \,. \end{cases}$$

The total photon propagator is now

$$\text{(17)} \qquad D_{\mu\nu}(x - x') = \langle T(\boldsymbol{a}_\mu(x), \boldsymbol{a}_\nu(x')) \rangle_0 + \langle T(\partial_\mu \Lambda(x), \partial'_\nu \Lambda(x')) \rangle_0 =$$

$$= \int_0^\infty \mathrm{d}a \varrho(a) \left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{a}\right) \Lambda_c(x - x', a) - iMn_\mu n_\nu \delta(x - x') -$$

$$- \int_0^\infty \mathrm{d}a \varrho_1(a) \partial_\mu \partial_\nu \Lambda_c(x - x', a) + iMn_\mu n_\nu \delta(x - x') =$$

$$= \int_0^\infty \mathrm{d}a \left\{ \varrho(a) \left(g_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{a}\right) - \varrho_1(a) \partial_\mu \partial_\nu \right\} \Lambda_c(x - x', a) \,.$$

Therefore,

$$\text{(18)} \qquad \partial^\mu D_{\mu\nu}(x - x') = \int_0^\infty \mathrm{d}a \left\{ \varrho(a) \partial_\nu \left(1 - \frac{\Box}{a}\right) - \varrho_1(a) \partial_\nu \Box \right\} \Lambda_c(x - x', a) =$$

$$= \partial_\nu \left\{ -i \int_0^\infty \mathrm{d}a \frac{\varrho(a)}{a} \delta(x - x') - i \int_0^\infty \mathrm{d}a \varrho_1(a) \delta(x - x') \right\} -$$

$$- \partial_\nu \int_0^\infty \mathrm{d}a \varrho_1(a) \cdot a \cdot \Lambda_c(x - x', a) = \partial_\nu D_c(x - x'),$$

where the relations (15) and (16) have been used and the n_μ is the time-like unit vector.

We can further derive a relation between the radiative correction of the vertex part and the *improper* self-energy part as follows:

$$\text{(19)} \qquad \partial^\mu_y \langle T(\boldsymbol{J}_\mu(y), \boldsymbol{I}(x), \bar{\boldsymbol{I}}(x')) \rangle_0 =$$

$$= e_0 \left\{ \langle T(\boldsymbol{I}(x), \bar{\boldsymbol{I}}(y)) \rangle_0 \delta(y - x') - \delta(x - y) \langle T(\boldsymbol{I}(y), \bar{\boldsymbol{I}}(x')) \rangle_0 \right\},$$

where

$$\text{(20)} \qquad \boldsymbol{I}(x) \equiv ie_0 \boldsymbol{A}_\mu(x) \gamma^\mu \boldsymbol{\psi}(x) + \delta m \boldsymbol{\psi}(x),$$

and

$$\delta(x_0 - x_0')[\boldsymbol{I}(x), \boldsymbol{J}_0(x')] = e_0 \delta(x - x') \boldsymbol{I}(x') \,. \tag{21}$$

The insertion of the expression

$$\begin{cases} \langle T(\boldsymbol{I}(x), \bar{\boldsymbol{I}}(x'), \boldsymbol{J}_\mu(y)) \rangle_0 = \dfrac{ie_0}{(2\pi)^8} \displaystyle\int \mathrm{d}^4 p \, \mathrm{d}^4 q \, \exp\,[ip(x - y)] \exp\,[iq(y - x')] \Lambda_\mu(p; q) \,, \\ \langle T(\boldsymbol{I}(x), \boldsymbol{I}(x')) \rangle_0 = \dfrac{i}{(2\pi)^4} \displaystyle\int \mathrm{d}^4 p \, \exp\,[ip(x - x')] \sum_0 (i\gamma p) \,, \end{cases} \tag{22}$$

gives

$$-i(p - q)^\mu \Lambda_\mu(p; q) = \sum\nolimits_0 (i\gamma p) - \sum\nolimits_0 (i\gamma q) \,. \tag{23}$$

An application of the equations (10) and (23) will be presented in a forthcoming paper.

* * *

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Polarization of Electrons from μ-e Decay.

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(ricevuto il 7 Maggio 1957)

The two-component neutrino theory of Lee and Yang [1] predicts that muons from π-μ decay should be completely polarized along their direction of motion, and the electrons from the decay of such muons should show an asymmetric distribution with respect to the muon path. This asymmetry has been found experimentally [2]. It is to be expected that, apart from exhibiting asymmetry in their angular distribution, the electrons will also be polarized. To examine this question, we have used the Hamiltonian, eq. (21) of Lee and Yang [1], for the process

$$\mu \to e + \nu + \bar{\nu}\,, \tag{1}$$

and calculated the probability of the muon to decay into a right-handed neutrino, its antiparticle and an electron of given spin orientation. The wave functions for a free Dirac particle with spin in positive (negative) z direction are with the choice of γ matrices of reference [1]:

$$u = \left(\frac{E+p_3}{2E}\right)^{\frac{1}{2}} \begin{pmatrix} 1 \\ \dfrac{p_1+ip_2}{E+p_3} \\ \dfrac{m}{E+p_3} \\ 0 \end{pmatrix}, \qquad v = \left(\frac{E+p_3}{2E}\right)^{\frac{1}{2}} \begin{pmatrix} 0 \\ \dfrac{m}{E+p_3} \\ -\dfrac{p_1-ip_2}{E+p_3} \\ 1 \end{pmatrix}, \tag{2}$$

[1] T. D. Lee and C. N. Yang: *Phys. Rev.*, **105**, 1671 (1957).
[2] R. L. Garwin, L. M. Lederman and M. Weinrich: *Phys. Rev.*, **105**, 1415 (1957).

and with spin in the direction θ, φ:

$$\psi = u \cos \tfrac{1}{2}\theta + v \sin \tfrac{1}{2}\theta \exp [i\varphi] . \tag{3}$$

Spin direction is defined in the rest system of the particle [3].

Calling ϑ = angle $(\boldsymbol{p}_e, \boldsymbol{s}_\mu)$, and (θ, φ) = direction of $\boldsymbol{s}_e$ with respect to $\boldsymbol{p}_e$ as z-axis and $(\boldsymbol{p}_e, \boldsymbol{s}_\mu)$ plane as xz plane, we find (with assumption of real coupling constants, and setting $m_e = 0$) a transition probability

$$w = m_\mu^5 (2\pi)^{-4} (3.2^6)^{-1} x^2 \mathrm{d}x \mathrm{d}\Omega_e \Big\{ (f_V^2 + f_A^2)[3 - 2x - (1 - 2x) \cos\vartheta \cos\theta] + \\ + 2 f_V f_A [(1 - 2x) \cos\vartheta - (3 - 2x) \cos\theta] + (f_V^2 - f_A^2) \sin\vartheta \sin\theta \cos\varphi \Big\} , \tag{4}$$

and a polarization

$$P = \frac{w(\theta, \varphi) - w(\pi - \theta, \varphi + \pi)}{w(\theta, \varphi) + w(\pi - \theta, \varphi + \pi)} = \\ = - \frac{[\xi(3 - 2x) + (1 - 2x) \cos\vartheta] \cos\theta - (1 - \xi^2)^{\frac{1}{2}} \sin\vartheta \sin\theta \cos\varphi}{3 - 2x + \xi(1 - 2x) \cos\vartheta} , \tag{5}$$

with $\xi = 2 f_V f_A (f_V^2 + f_A^2)^{-1}$, $x = p_e / p_{e\,\max}$. The square root is taken to be of positive sign, which is the case for $f_V^2 > f_A^2$. $|P|$ has a maximum for

$$\varphi_{\max} = \pi , \qquad \operatorname{tg} \theta_{\max} = (1 - \xi^2)^{\frac{1}{2}} \sin\vartheta [\xi(3 - 2x) + (1 - 2x) \cos\vartheta]^{-1} , \tag{6}$$

which determines the « spin direction » of the electron, and the corresponding maximum polarisation is

$$P_{\max} = - \frac{\{[\xi(3 - 2x) + (1 - 2x) \cos\vartheta]^2 + (1 - \xi^2) \sin^2\vartheta\}^{\frac{1}{2}}}{3 - 2x + \xi(1 - 2x) \cos\vartheta} . \tag{7}$$

This may reach 100%, e.g. at $x = 1$ and any ϑ. One finds that the spin of the electron points predominantly backward (forward) with respect to its motion if

$$\xi(3 - 2x) + (1 - 2x) \cos\vartheta \gtrless 0 . \tag{8}$$

If such polarised electrons pass through matter, they will emit bremsstrahlung which is circularly polarized. This fact may serve to verify the electron polarization experimentally, and a theoretical investigation of this aspect of bremsstrahlung is in progress.

* * *

I benefited by discussions with Dr. J. Holt, and by stimulation of Mr. P. K. Kabir.

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(3) H. A. Tolhoek and S. R. deGroot: *Physica*, **17**, 1 (1951).

Comparison between the Proton Energy Spectra of (n, p) and (d, p) Reactions.

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(ricevuto l'11 Maggio 1957)

In the proton energy spectra of (n, p) reactions obtained with 14 MeV neutrons Colli *et al.* ([1-3]) found deviations from a smooth shape. In the discussion of these results it was suggested that these deviations have some relation to giant resonances as were found in neutron cross sections and were explained by the optical model (Colli *et al.* ([4])).

Confirmation of this relation can be obtained by comparing these proton spectra with the proton spectra of (d, p) reactions as measured by the M.I.T. group. We compared reactions with the same product nucleus viz. ^{24}Mg(n, p)^{24}Na with ^{23}Na(d, p)^{24}Na ([5]) and ^{28}Si(n, p)^{28}Al with ^{27}Al(d, p)^{28}Al ([6]) and tried to find out if relative maxima in the (n, p) spectra correspond with giant resonances

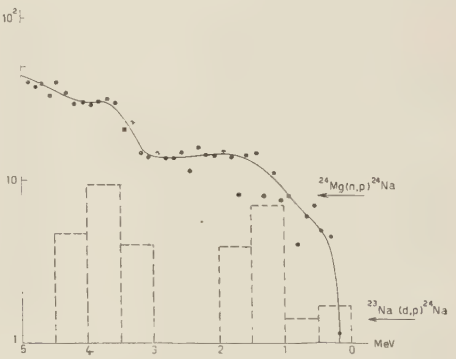

Fig. 1. – Comparison between the proton energy spectra of ^{24}Mg(n, p)^{24}Na and ^{23}Na(d, p)^{24}Na; the spectra are plotted as functions of the excitation energy of the product nucleus.

in the (d, p) spectra. To obtain these giant resonances we had to average the sharp resonances of the (d, p) spectra over suitably chosen energy intervals (0.5 MeV). For the comparison the spectra must be plotted as a function of the excitation energy of the product

([1]) L. Colli and U. Facchini: *Nuovo Cimento*, **4**, 671 (1956).

([2]) C. Badoni, L. Colli and U. Facchini: *Nuovo Cimento*, **4**, 1618 (1956).

([3]) L. Colli and U. Facchini: *Nuovo Cimento*, **5**, 309 (1957).

([4]) L. Colli, U. Facchini and S. Micheletti: *Nuovo Cimento*, **5**, 502 (1957).

([5]) A. Sperduto and W. W. Buechner: *Phys. Rev.*, **88**, 574 (1952).

([6]) W. W. Buechner, M. Mazari and A. Sperduto: *Phys. Rev.*, **101**, 188 (1956).

nucleus, which can be computed easily from the energy of incident and outgoing particles, the Q-values and the recoil energies of the compound and the product nuclei.

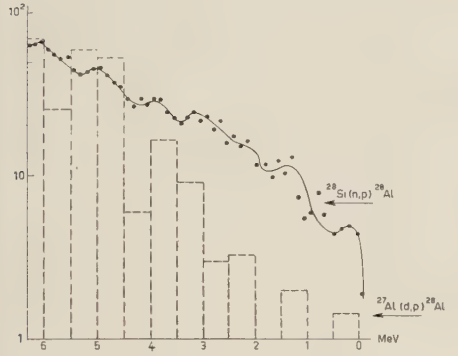

Fig. 2. – Comparison between the proton energy spectra of ^{28}Si(n, p)^{28}Al and ^{27}Al(d, p)^{28}Al; the spectra are plotted as functions of the excitation energy of the product nucleus.

In Figs. 1 and 2 the curves are the (n, p) spectra and the histograms are the results of the averaging of the (d, p) spectra. In Fig. 1 the correspondence is clear. From Fig. 2 can be seen that most of the maxima in the (n, p) spectra occur also in the averaged (d, p) spectrum. From this can be concluded that the deviations in the proton energy spectra of (n, p) reactions have the same origin as the giant resonances in the (d, p) spectra, although there is a large difference in reaction mechanism e.g. in the contribution by compound nucleus formation.

Pion-Nucleon S-Waves and K-mesons.

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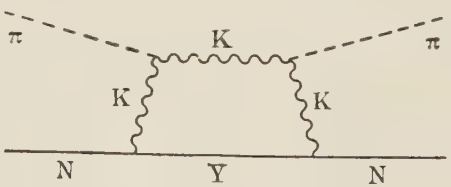
(ricevuto l'11 Maggio 1957)

The hypothesis that the pion interacts with the nucleon through a heavy meson field is in principle apt to furnish a physical basis for a dynamical interpretation of a non local pion-nucleon interaction (1). If K-mesons are taken to represent the internal meson field the dimensions of the nucleon source for π-mesons is of the order of magnitude of that requested in PS-PV static theory to explain P wave pion-nucleon scattering. It may be interesting to examine the implications of the dynamical model of the extended source proposed in (I) in other phenomena also, as for example pion-nucleon S wave scattering.

In the S-matrix theory the lowest orders in the coupling constants in which S-wave pion-nucleon scattering can be obtained from the interaction lagrangian (29+30) of (I) are the fourth and the sixth one. The sixth order term is equivalent to the usual second-order term of the local theory, each vertex being substituted by a product of three propagators $\Delta_K^c \Delta_K^c S_Y^c$. The fourth order term is a direct consequence of the lagrangian (29) of (I) and may be symbolically represented by the diagram:



This process has no correspondent in the usual local theory. We find that the contribution of these two processes to the S-matrix are opposite in sign.

In order to evaluate the order of magnitude of the corresponding scattering matrix at low energy, we proceed as follows.

(1) P. Budini and L. Fonda: *Nuovo Cimento*, **5**, 306 (1957); P. Budini and L. Fonda: *Nuovo Cimento*, **5**, 666 (1957), this will be quoted in the following as (I).

Starting from the action integral corresponding to the equation (37) of (I), which describes the interaction of the pion with the nucleon through the K field (*):

$$(1)\qquad W=\int\{L_0^N(x)+L_0^\pi(x)\}\,\mathrm{d}^4x-$$

$$-iG_{K\pi}M_N\int \mathrm{d}^4(x'x''x''')2i\Delta_K^c(x'-x''')\Delta_K^c(x'''-x'')\big[g_{K\Sigma}^2\mu_\Sigma\Gamma_\Sigma^c(x'-x'')-g_{K\Lambda}^2\mu_\Lambda\Delta_\Lambda^c(x'-x'')\big]\cdot$$

$$\cdot N(x')\gamma_5\boldsymbol{\tau}N(x'')\boldsymbol{\pi}(x''')+$$

$$+G_{K\pi}^2M_N^2\int \mathrm{d}^4(x'x''x'''x'''')2i\Delta_K^c(x'-x'''')\Delta_K^c(x''''-x'')\Delta_K^c(x''-x''')\pi_j(x'')\bar{N}(x''')\cdot$$

$$\cdot\big\{g_{K\Lambda}^2\tau_j\tau_i\nabla'''\Delta_\Lambda^c(x'-x''')+g_{K\Sigma}^2(4\delta_{ij}-\tau_j\tau_i)\nabla'''\Delta_\Sigma^c(x'-x''')\big\}N(x')\pi_i(x'''')\,,$$

we have evaluated the kernels in the second and third term in the non relativistic low energy limit. This approximation will be justified for the low energy processes which we are going to consider. We obtain thus a local action integral whose corresponding lagrangian is:

$$(2)\qquad L(x)=L_0^N(x)+L_0^\pi(x)-iG_{K\pi}M_N[g_{K\Sigma}^2\mu_\Sigma I_\Sigma^\Delta-g_{K\Lambda}^2\mu_\Lambda I_\Lambda^\Delta]\bar{N}(x)\gamma_5\boldsymbol{\tau}N(x)\boldsymbol{\pi}(x)+$$

$$+G_{K\pi}^2M_N^2[g_{K\Lambda}^2I_\Lambda^\square+3g_{K\Sigma}^2I_\Sigma^\square]\bar{N}(x)N(x)\boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)\,,$$

with I_y^Δ and $I_y^\square$ given by:

$$(3)\qquad I_y^\Delta=\lim_{p_1=p_2=0}\frac{2i}{(2\pi)^4}\int \mathrm{d}^4k\big\{[(p_1-k)^2+\mu_k^2][(p_2-k)^2+\mu_K^2][k^2+\mu_y^2]\big\}^{-1}\,,$$

$$(4)\qquad I_y^\square=\lim_{p_1=p_2=p_3=0}\frac{2i}{(2\pi)^4}\cdot$$

$$\cdot\int \mathrm{d}^4k\,\frac{ik}{[(p_1-k)^2+\mu_K^2][(p_2-p_3+k)^2+\mu_K^2][(p_3-k)^2+\mu_K^2][k^2+\mu_y^2]}\,.$$

On the hamiltonian obtained from this lagrangian we apply the Dyson-Foldy transformation. Retaining only terms no more than quadratic in $G_{K\pi}$, we obtain ultimately:

$$(5)\qquad H'(x)=H_0^N(x)+H_0^\pi(x)+\frac{G_{K\pi}I_\Delta}{2}\bar{N}(x)\gamma_5\gamma_\mu\frac{\partial\boldsymbol{\pi}(x)}{\partial x_\mu}\boldsymbol{\tau}N(x)+$$

$$+\left(\frac{G_{K\pi}I_\Delta}{2}\right)^2\bar{N}(x)\gamma_5\boldsymbol{P}(x)\cdot\boldsymbol{\tau}\wedge\boldsymbol{\pi}(x)N(x)+\frac{G_{K\pi}^2M_N}{2}(I_\Delta^2-2M_NI_\square)N(x)N(x)\boldsymbol{\pi}(x)^2\,,$$

(*) The symbols are slightly changed:

$$\bar{N}=\tilde{N}^{*T}\qquad G_{K\pi}=g_\pi/M_N\,,\quad g_{K\Lambda}^2=g_5^2=g_5'^2\,,\qquad g_{K\Sigma}^2=g_6^2=g_6'^2\,,$$

we have explicitly assumed for semplicity the equality of g_5^2 with $g_5'^2$ and g_6^2 with $g_6'^2$.

where $\boldsymbol{P}(x)$ is the momentum conjugate to the field variable $\boldsymbol{\pi}(x)$ (*). I_Δ and $I_\square$ are given by:

$$I_\Delta = g^2_{K\Sigma}\mu_\Sigma I^\Delta_\Sigma - g^2_{K\Lambda}\mu_\Lambda I^\Delta_\Lambda , \qquad I_\square = g^2_{K\Lambda} I^\square_\Lambda + 3 g^2_{K\Sigma} I^\square_\Sigma . \tag{6}$$

Being I^2_Δ and $I_\square$ positive quantities it is manifest that the two processes mentioned above give interfering contribution to the term $NN\boldsymbol{\pi}^2$. It is known that this term has been lengthy discussed in the literature in order to bring the theoretical calculations in agreement with the experimental data on pion-nucleon S-waves. In general it has to be strongly damped from its value as deduced from the γ_5-coupling. In order to test if the reduction produced by the interference due to the presence of K-mesons is by itself of the magnitude order requested by experiments, we put:

$$\frac{G^2_{K\pi} M_N}{2} (I^2_\Delta - 2 M_N I_\square) = l_0 \frac{G^2_{K\pi} M_N}{2} I^2_\Delta , \tag{7}$$

with $l_0 = 0.03$.

We identify further the coefficient of the term $\bar{N}\gamma_5\gamma_\mu(\partial\boldsymbol{\pi}/\partial x_\mu)\boldsymbol{\tau}N$ with the known coupling constant which gives agreement with the experimental results on P waves:

$$\frac{G_{K\pi} I_\Delta}{2} = \frac{f}{\mu_\pi}\sqrt{4\pi} . \tag{8}$$

From these conditions we deduced the numerical relations between the coupling constants involved as given in Table I, taking for f^2 the value 0.1. The last row gives the ratio $g^2_{K\Sigma}/g^2_{K\Lambda}$. The values of $g^2_{K\Sigma}$ and $g^2_{K\Sigma}/g^2_{K\Lambda}$ are independent of the choice of f^2, since they are a consequence of the condition (7).

It is to be remarked that because of the approximation used and having neglected all renormalization effects, the results are only meaningful as order of magnitude. They are not in disagreement with the experimental data so far known.

TABLE I.

$g^2_{K\Lambda}/4\pi$	1	2.5	5	12.5
$g^2_{K\Sigma}/4\pi$	22	25	30	43
$G^2_{K\pi}/4\pi$	0.72	0.61	0.50	0.33
$g^2_{K\Sigma}/g^2_{K\Lambda}$	22	10	6	3.4

(*) If a direct pion-nucleon coupling is also admitted, then one has only to substitute $G_{K\pi} I_{\Delta/2}$: with $G_{K\pi} I_{\Delta/2} + f'\sqrt{4\pi}/\mu_\pi$ where f' is the coupling constant for the direct interaction. This introduces a further parameter into the theory.

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A Systematization of Weak Interactions.

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(ricevuto il 19 Maggio 1957)

Weak interactions between elementary particles have coupling constants of the same order of magnitude, but their selection rules are rather curious. Particularly, it is quite difficult to explain the absences of π-e decay and K_{e2}: Recent observations show that the branching ratio $(\pi \to e+\nu)/(\pi \to \mu+\nu)$ is less than 10^{-5}, and probably this fact cannot be explained by the postulate of universal Fermi interactions [1].

Now recently experiments proved the non-conservation of parity in weak interactions [2,3], and Lee and Yang's two-component theory of the neutrino [4] seems very reliable. If their theory is accepted, there exist only the « right-handed screw » neutrinos,

$$\psi_\nu = \frac{1-\gamma_5}{2}\boldsymbol{\psi}_\nu ,$$

in our world ($\boldsymbol{\psi}_\nu$ denotes Dirac neutrinos).

However, the « left-handed screw » neutrinos,

$$\psi'_\nu = \frac{1+\gamma_5}{2}\boldsymbol{\psi}_\nu ,$$

should have the same title to exist from reasons of symmetry. Hence we suppose the other (fantastic) world where only ψ'_ν would exist. Here we consider a transformation from our world to the other one:

$$\psi_\nu = \frac{1-\gamma_5}{2}\boldsymbol{\psi}_\nu \to \frac{1-\gamma_5}{2}\cdot\gamma_4\boldsymbol{\psi}_\nu = \gamma_4\psi'_\nu ,$$

under which we require the form-invariance of the weak interaction Lagrangian (*). The transformation characters of other quantities are assigned as follows.

electron: $\psi_e \to \gamma_4\gamma_5\psi_e$

muon: $\psi_\mu \to \gamma_4\psi_\mu$

nucleon: $\psi_N \to \gamma_4\gamma_5\psi_N$

hyperons: $\psi_Y \to \varepsilon_Y\gamma_4\gamma_5\psi_Y$ (spin $\frac{1}{2}$)

(1) e.g. S. B. Treiman and H. W. Wyld jr.: *Phys. Rev.*, **101**, 1552 (1956).

(2) C. S. Wu *et al.*: *Phys. Rev.*, **105**, 1413 (1957).

(3) R. L. Garwin, L. M. Lederman and M. Weinrich: *Phys. Rev.*, **105**, 1415 (1957).

(4) T. D. Lee and C. N. Yang: *Phys. Rev.*, **105**, 1671 (1957); A. Abashian *et al.*: *Phys. Rev.*, **105**, 1927 (1957).

(*) In order to avoid some ambiguity in the judgement of form-invariance, we always adopt the covariant forms of the minimum use of γ_5 (or $\varepsilon_{\mu\nu\sigma\tau}$) among equivalent expressions of coupling types.

(ε_Y is the relative parity multiplied by $(-1)^\eta$ (η is the strangeness)).

Scalars and vectors: invariant.

From the above assumptions it immediately follows that derivative coupling and μ-e coupling are forbidden. For Fermi interactions the allowed coupling types are as follows.

β-decay $(\bar{N}N)(\bar{e}\nu)$: S, V and T,

μ-capture $(\bar{N}N)(\bar{\nu}\mu)$: P and V,

μ-decay $(\bar{\nu}\mu)(\bar{e}\nu)$: V.

These are consistent with recent experiments ([5,3]). If Λ has the same parity as N,

$(\bar{\Lambda}N)(\bar{e}\nu)$: P and A,

$(\bar{\Lambda}N)(\bar{\mu}\nu)$: S, A and T.

The other types of interactions are:

$\pi\cdot(\bar{\nu}e)$, $K\cdot(\bar{\nu}e)$ and $K\cdot\pi\cdot(\bar{\nu}e)$: forbidden

$\pi\cdot(\bar{\nu}\mu)$, $K\cdot(\bar{\nu}\mu)$ and $K\cdot\pi\cdot(\bar{\nu}\mu)$: allowed

$K\cdot\pi\cdot\pi$ and $K\cdot\pi\cdot\pi\cdot\pi$: allowed.

Meson decays via virtual baryon pairs are:

π-e decay: forbidden,

K_{e2}: almost forbidden.

other modes (except $K\to 2\pi$): allowed (especially notice that K_{e3} is allowed) provided parity is conserved in strong interactions. Thus μ-e coupling, π-e decay and K_{e2} could be forbidden.

Detailed discussions of the present scheme will be published elsewhere.

([5]) M. L. Good and E. J. Lauer: *Phys. Rev.*, **105**, 213 (1957). Probably V in β-decay is not small provided time reversal is violated.

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Evidence of a Second Maximum in the Cross Section for Fast Photoneutron Emission in Cr, As, I, Ta.

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Istituto Nazionale di Fisica Nucleare - Sezione di Torino

(ricevuto il 23 Maggio 1957)

Continuing the experiments on the fast neutron component in photonuclear reactions, we have measured the excitation curves for 25 elements with atomic number ranging from 13 to 82.

The experimental arrangement has been described in a previous paper [1]; the neutron counter is a (n, p) threshold detector, whose sensitivity is roughly constant from 5 to 14 MeV. The excitation curves were measured from the fast neutron threshold up to 30 MeV, using the X-ray beam from a B.B.C. Betatron.

Among the 25 elements examined in the present work four particularly interesting cases were found: Cr, As, I, Ta. The yield curves are given in Fig. 1. Analyzing the above curves with a method of successive approximation, we find the existence of two separate maxima in the cross section for fast neutron emission. The experimental ratios R between the area of the second maximum over the area of the first one are given for each element in Table I.

Table I.

Element	R
Cr	0.7 ± 0.14
As	0.6 ± 0.12
I	0.4 ± 0.08
Ta	0.5 ± 0.1

We observe, incidentally, that the above elements are isotopically pure, except chromium where, however, the percentage of ^{52}Cr is 83.7%. Therefore the absence of a second maximum in the other measured elements is not a proof that the second résonance is very rare and due to a particular structure of the above elements, but it may be present also in many other elements, being masked generally by the isotopic complexity.

In the case of tantalum we have measured the total (γ, n) yield with the method of residual activity and we found reliable evidence of a second maximum also in this case. Therefore,

[1] F. Ferrero, A. O. Hanson, R. Malvano and C. Tribuno: *Nuovo Cimento*, **4**, 418 (1956).

at least for tantalum the assumption that the second resonance could be attributed to the build up of a $(\gamma, 2n)$ process is not only in disagreement with the experimental data but is also in contradiction with the essentially evaporative nature of such a type of photodisintegration.

It is too early to try any detailed explanation of the effect. We may observe, however, its resemblance with the two separate maxima for the (γ, n) and (γ, p) cross sections observed in ^{25}Mg [2], ^{100}Mo [3], ^{40}Ar [4].

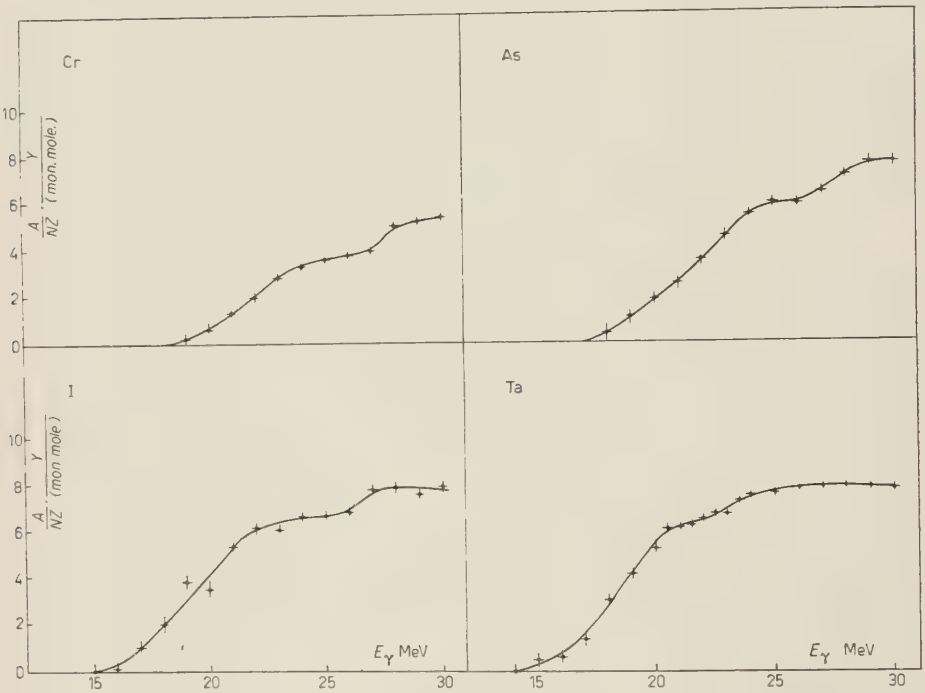

Fig. 1.

* * *

We are going on with the experiments, extending them to other elements and trying a different technique for the fast neutron detector. We like to acknowledge very much Proff. A. O. Hanson. L. Katz, E. Toms for their interest in the present work and Proff. G. Wataghin, R. Deaglio for their steady encouragement.

(2) L. Katz, R. N. H. Haslam, J. Goldemberg and J. G. V. Taylor: *Canad. Journ. of Phys.*, **32**, 580 (1954).

(3) W. A. Butler and G. N. Almy: *Phys. Rev.*, **91**, 58 (1953).

(4) D. McPherson, E. Pederson and L. Katz: *Canad. Journ. of Phys.*, **32**, 593 (1954).

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On the Variation of the Diamagnetic Susceptibility of Water due to Temperature.

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(ricevuto il 24 Maggio 1957)

The diamagnetic susceptibility of water is a very important constant since a large number of ionic susceptibilities of metals are measured in aqueous solution. The experimental results concerning the variation of the diamagnetic susceptibility of water with temperature differ from each other [1]. The best values seem to be the values given by Cabrera and Fahlenbrach [2]. The first who tried to calculate theoretically the variation of diamagnetic susceptibility of water with temperature were Honda and Shimuzu [3]. Honda and Shimuzu used the statistical theory of the atom and some additional assumptions. Since at that time there were no accurate approximate solutions of the Thomas-Fermi equation the results of Honda and Shimuzu do not agree well with the data given by Cabrera and Fahlenbrach. In this paper we shall calculate in another way the variation of the diamagnetic susceptibility of water with temperature using the approximate solution for a free neutral atom proposed by the author [4]:

$$(1) \qquad \begin{cases} y(x) = (1 + ax + bx^2)^{-\frac{3}{2}}, \\ a = 0.7105, \quad b = 0.03919. \end{cases}$$

According to Honda and Shimuzu we assume that the water molecule is a neutral atom and the electrons having a distance from the nucleus greater than the mean atomic radius are assumed to become free electrons. Then the expansion of volume due to a rise of temperature causes the increase of diamagnetic susceptibility by the increase of bound electrons at a cost of free electrons. The diamagnetism of water molecules is mainly due to the electrons in O^{--} namely to the 10 electrons around the oxygen nucleus (*). On the assumption of Honda and Shimuzu the number

(1) E.g. compare the data of R. N. Mathur: *Ind. Journ. Phys.*, **15**, 207 (1931), with the data of B. Cabrera and H. Fahlenbrach: *Zeits. Phys.*, **82**, 759 (1933).

(2) See reference (1).

(3) K. Honda and Y. Shimuzu: *Nature*, **132**, 565 (1933); **135**, 108 (1935); *Sci. Rep.*, **25**, 939; Y. Shimuzu: *Sci. Rep.* (1933).

(4) T. Tietz: *Nuovo Cimento*, **4**, 1192 (1956).

(*) This assumption is discussed in the mentioned paper by Honda and Shimuzu in more details.

TABLE I.

Temp.	Density (ϱ_t)	α_t from equation (4)	χ_{t^0}/χ_{20^0} HONDA and SHIMUZU	χ_{t^0}/χ_{20^0} from our approximation	χ_{t^0}/χ_{20^0} from LATTER's approximation	χ_{t^0}/χ_{20^0} experimental data of CABRERA and FAHLENBRACH
90°	0.965 3	0.823 5	1.027 2	1.006 2	1.006 5	1.008 1
70°	0.977 8	0.829 3	1.016 9	1.003 8	1.004 0	1.006 1
60°	0.983 2	0.831 7	1.012 3	1.002 8	1.002 9	1.004 6
50°	0.988 1	0.834 0	1.008 3	1.001 9	1.002 0	1.003 3
40°	0.992 2	0.835 8	1.005 0	1.001 1	1.001 2	1.002 4
30°	0.995 7	0.837 4	1.002 0	1.000 5	1.000 5	1.001 2
20°	0.998 2	0.838 5	1.000 0	1.000 0	1.000 0	1.000 0
10°	0.999 7	0.839 2	0.998 7	0.999 7	0.999 7	0.999 2
4°	1.000 0	0.839 4	0.998 4	0.999 6	0.999 6	0.998 3

of free electrons per water molecule α_t is given by:

$$\frac{\alpha_t}{Z} = \int_{x_t}^{\infty} y^{\frac{3}{2}} x^{\frac{1}{2}} \, \mathrm{d}x \,. \tag{2}$$

In the last formula y is the solution of the Thomas-Fermi equation for a free neutral atom and Z is the number of electrons around the oxygen nucleus. Taking into consideration the Thomas-Fermi equation for a free neutral atom and the boundary condition for y we see that:

$$y(x_t) - x_t \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=x_t} = \frac{\alpha_t}{Z} \,. \tag{3}$$

Substituting equation (1) into the last formula we obtain in our case the number of free electrons per water molecule at the temperature t.

$$\frac{\alpha_t}{Z} = \frac{2 + 5ax_t + 8bx_t^2}{2\,(1 + ax_t + bx_t^2)^{\frac{5}{2}}} \,. \tag{4}$$

Using the mass of the water molecule $M = 2.99 \cdot 10^{-23}$ we have for x_t the following relation:

$$\begin{cases} x_t = \dfrac{1}{\mu} \sqrt[3]{\dfrac{3M}{4\pi\varrho_t}} \,, \\ \mu = 0.88534\, a_0 / Z^{\frac{1}{3}} \,. \end{cases} \tag{5}$$

In the last formula ϱ_t is the density of water at the temperature t and a_0 is the Bohr radius. Formulas (4) and (5) allow us to calculate the number α_t of free electrons of the water molecule at the temperature t. The results are given in Table I. In order to calculate the variation of diamagnetic susceptibility of water χ_t with temperature we start with the formula for the diamagnetic susceptibility given by GOMBAS [5].

$$\chi_{t^0} = -\frac{9.43 \cdot 10^{-6}}{Z^{\frac{5}{3}} W} \cdot \left(1 - 3.01 \frac{\alpha_t}{Z} + 3.92 \frac{\alpha_t^2}{Z^2}\right) (Z - \alpha_t) \ \mathrm{cm}^3 \,, \tag{6}$$

where $W = 18.016$ is the molecular weight of water. Formulas (4), (5) and (6)

[5] P. GOMBAS: *Zeits. Phys.*, **87**, 57 (1933).
[6] R. LATTER: *Phys. Rev.*, **99**, 510 (1955).

allow us to calculate the diamagnetic susceptibility χ_{t^0} at temperature t. In Table I we have compared our values χ_t^0/χ_{20^0} with the values of Honda and Shimuzu, and the values of Cabrera and Fahlenbrach. Table I contains also the values of χ_{t^0}/χ_{20^0} calculated with the best approximate solution of the Thomas-Fermi equation for the free neutral atom due to LATTER (6).

Table I shows that our results for χ_{t^0}/χ_{20^0} differ very little from the results calculated with Latter's approximation. Our results are in better agreement with the results of Cabrera and Fahlenbrach than the results of Honda and Shimuzu.

In our case we obtain for χ_{20^0} the value:

$$\chi_{20^0} = -0.7990 \cdot 10^{-6} \text{ cm}^3 .$$

The observed value of χ_{t^0} at room temperature is $-0.720 \cdot 10^{-6}$ cm^3 and the value of χ_{20^0} calculated with Latter's approximation is $-0.7624 \cdot 10^{-6}$ cm^3. The values of χ_t and χ_{t^0}/χ_{20^0} given by Honda and Shimuzu (7) should be multiplied by a factor 1.045 because the coefficient 3.1 in their equation (10) should be replaced by 3.24 according to the paper of HIRONE (8).

* * *

The author is indebted to Professor J. WIŚNIEWSKI for his interest in this note.

(7) K. HONDA and Y. SHIMUZU: *Sci. Rep.*, **25**, 939 (1937).

(8) T. HIRONE: *Sci. Rep.*, **24**, 264 (1935).

Time Reversal and the Decay of the μ-Meson.

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(ricevuto il 1° Giugno 1957)

If it be assumed that the μ-meson decays into an electron, a neutrino and an antineutrino, and that the interaction is of « two-component » type, not involving derivatives, the electron distribution from the decay of a completely polarized μ-meson at rest is

$$\mathrm{d}N = f(x)x^2\,\mathrm{d}x\,\mathrm{d}\Omega/2\pi\,, \tag{1}$$

where

$$f(x) = 3 - 2x + 3\zeta(1 - v^2/c^2)^{\frac{1}{2}} + \xi\cos\theta(v/c - 2x)\,.$$

Here p, v, m are the momentum, velocity and mass of the electron, θ is the angle between p and the spin of the μ-meson, $\mathrm{d}\Omega$ is the differential solid angle into which the electron is emitted and $x = 2p/M$, where M is the mass of the μ-meson. The ratio m/M is neglected, but not m/p, since we shall be concerned with slow electrons. ξ and ζ are functions of the coupling constants

$$\xi = 2\,\mathrm{Re}\,f_A^* f_V(|f_A|^2 + |f_V|^2)^{-1}\,,$$

$$\zeta = (|f_A|^2 - |f_V|^2)(|f_A|^2 + |f_V|^2)^{-1}\,.$$

This formula was given by Lee and Yang [1], in the approximation $v/c \to 1$, and has been confirmed experimentally [2], with $\xi = 0.87 \pm 0.12$.

We can form one other combination of the coupling constants which is independent of the Pursey [3] transformation, and thus can in principle be measured experimentally. This is $\eta = 2\,\mathrm{Im}\,f_A^* f_V(|f_A|^2 + |f_V|^2)^{-1}$, which vanishes if and only if the interaction is invariant under time reversal. It satisfies the relation

$$\xi^2 + \eta^2 + \zeta^2 = 1\,, \tag{2}$$

(1) T. D. Lee and C. N. Yang: *Phys. Rev.*, **105**, 1671 (1957).

(2) See D. H. Wilkinson (to appear in *Nuovo Cimento*) for a review of the experiments.

(3) D. Pursey (preprint, *Invariance Properties of Fermi Interactions*, April 1957). This transformation has also been considered by W. Pauli (private communication).

leading to the restriction $\eta < 0.5$ when we use the experimental mean value of ξ. An accurate determination of the electron spectrum below 2 MeV would give a value for ζ, from (1), and so lower the upper bound for η. It is unlikely that the accuracy of the experimental value for ξ can be improved without a great deal of work, and hence we cannot expect to be able to discover whether η differs from zero by this method.

The only other type of measurement we can make on this decay is to determine the polarization of the emitted electron. If we measure the longitudinal component of the electron spin, we obtain [4]

$$(3) \qquad \langle \sigma_L \rangle = \frac{I_+ - I_-}{I_+ + I_-} = -[\xi(3v/c - 2x) + \cos\theta\big(1 + \zeta(1 - v^2/c^2)^{\frac{1}{2}} - 2x\big]/f(x) \rightarrow$$

$$\rightarrow -[\xi(3 - 2x) + \cos\theta(1 - 2x)][(3 - 2x) + \xi\cos\theta(1 - 2x)]^{-1} \quad \text{for } v/c \rightarrow 1.$$

Thus we see that a measurement of the longitudinal spin would afford a valuable check on our assumptions, but if they are accepted, it would provide no new information. A similar calculation of the expectation of the spin component normal to the plane containing the spin of the μ-meson and the electron momentum gives $\langle \sigma_T \rangle = (1 - v^2/c^2)^{\frac{1}{2}}\eta v \sin\theta/cf(x)$. If the spin is to be detected in the only way currently available, differential Mott scattering, we are interested in the relative intensity of the eigenstates of $\beta\sigma_T$, an operator which commutes with the Hamiltonian, rather than those of σ_T (which does not). We obtain

$$\frac{I_+ - I_-}{I_+ + I_-} = \eta v \sin\theta/cf(x) \,.$$

It is only the slowest electrons, with $v/c \sim 0.6$, whose quantum state can be detected in this way; their relative intensity is of the order of a few times $(m/M)^3$, or say, one in a million. It appears, however, that the effect could probably be detected (provided η is not too small), with magnetic momentum selection and fast coincidence techniques, if we had considerably more intense μ-meson beams than those available at present.

* * *

I am grateful to several colleagues, and particularly to Dr. S. D. Bloom, for discussions of the experimental possibilities.

(4) While this letter was being written, I was informed by Professor Frisch that Dr. Überall of Liverpool has also derived this formula. His manuscript also gives the expectation of β times the transverse spin in the plane containing the spin of the μ-meson and the electron momentum, a quantity involving ζ.

The Optical Absorption of $Ni^{2+} \cdot 6H_2O$ at Low Temperatures.

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(ricevuto il 3 Giugno 1957)

Information concerning the electronic configurations, level positions and ligand field symmetry of complex ions can be obtained from their optical absorption, as a useful complement and alternative to paramagnetic resonance methods [1].

At room temperature the thermal agitation of the lattice obscures the detailed form of this absorption, and it is advantageous to study the optical behaviour at low temperatures. Exploratory work has been initiated with the hexaquo complexes of Ni^{2+}, Co^{2+} and Mn^{2+}, at the temperatures of liquid helium (~ 4 °K) and liquid hydrogen (~ 20 °K).

An interesting phenomenon occurs with crystals of nickel fluosilicate ($NiSiF_6 \cdot 6H_2O$). On cooling the crystal to 20 °K and 4 °K the main band in the visible shifts its peak from 25 500 cm^{-1} at room temperature to 26 500 cm^{-1}, and in addition shows several sharp lines on its long-wavelength side. The effect was reproduced with three different crystals, thereby excluding instrumental effects.

The lines are sharp just at the beginning of the band. At 4 °K the first pattern of lines at 24 350 cm^{-1} is composed of, first a relatively faint line, then two stronger lines of about equal intensity, the spacing of these three being fairly even (roughly 60 cm^{-1}). Typical half-widths of the lines are about 40 cm^{-1}. About 170 cm^{-1} further is a doublet, then a complex pattern, apparently of four lines. On going further towards the maximum of the main absorption band the lines become blurred into narrow bands, in which it is difficult to separate the components.

The observed structure shows no appreciable change between 4 °K and 20 °K.

Other narrow bands occur in the visible at 19 350 cm^{-1}, and at 16 380, 16 260, 15 930 and 15 750 cm^{-1} (the differences are more accurate than the absolute values). There are also fainter narrow bands in the 16 000 cm^{-1} region which are still being investigated.

One may try and interpret the absorption spectrum in terms of ligand field theory. The ground state of $Ni^{2+} \cdot 6H_2O$ is $^3A_2$. One possibility, in keeping with the calculations of Tanabe and Sugano [2], and of Orgel [3], is

[1] A. Abragam and M. H. L. Pryce: *Proc. Roy. Soc.*, A **206**, 173 (1951).

[2] Y. Tanabe and S. Sugano: *Journ. Phys. Soc. Jap.*, **9**, 753 (1954).

[3] L. E. Orgel: *Journ. Chem. Soc.*, **4**, 4756 (1952).

that the sharp lines around 24 300 cm^{-1} are associated with the inter-system ([4,5]) combination $^3A_2 \rightarrow {}^1A_1$, which may be enhanced by intermixing ([6]) of $^1A_1$ with the nearby $^3T_1$ (which possess ([7]) a common representation (A_1) of the octahedral symmetry group). The energy difference $^3A_2 - {}^1A_1$ is essentially independent of the crystal field parameter, and so is unaffected by crystal field fluctuations, so that sharp lines are to be expected, instead of the more usual bands.

According to Tanabe and Sugano's theoretical estimates, the intensity of such intersystem combinations, which arise through electric dipole transitions, coupled with the excitation of a hemihedral (odd) vibration to give the necessary change of parity, corresponds to an oscillator strength between 10^{-6} and 10^{-8}. Assuming that the observed lines are superposed on a background of continuous absorption it is possible to estimate their oscillator strength from a densitometric analysis of the plates. The result, for the intense lines at 24 350 cm^{-1}, is $3 \cdot 10^{-3}$, which is within the theoretical limits.

The calculated position of $^1A_1$ is considerably nearer to $^3T_1$ than is observed, assuming that the theoretical $^3T_1$ position corresponds to the peak of the main absorption band ([2]). This discrepancy may be due to a resonance effect connected with the intermixing already mentioned; or to the temperature shift of the $^3T_1$ level, while the $^1A_1$ level is independent of temperature.

The assignment $^3A_2 \rightarrow {}^1A_1$ is not the only possible explanation for the observed lines. One might alternatively interpret them as due to vibrational structure associated with the $^3A_2 \rightarrow {}^3T_1$ transition, responsible for the main absorption band. This does not require us to postulate an intersystem combination. On the other hand, the following remarks argue against this hypothesis.

It is commonly assumed that the ionic radius of the central ion depends on the number ([8]) of electrons in the two possible types of sublevel, $d\varepsilon$ and $d\gamma$. The $^3A_2 \rightarrow {}^3T_1$ transition involves the jump by two electrons from $d\varepsilon$ to $d\gamma$, and thus is likely to produce a large change in ionic radius. To discuss the degree of excitation of vibrations in the transition one has to introduce the concept, familiar in molecular spectroscopy, of the potential energy as function of the configurational co-ordinates. According to what we have just said, it is reasonable to assume that in the transition $^3A_2 \rightarrow {}^3T_1$ the potential energy curve will be different in the initial and final levels, as regards the equilibrium position. It would thus seem probable that the transition would appreciably excite only vibrational levels with high quantum numbers, whose energies are closely packed, so giving an unresolved broad band.

On the other hand the $^1A_1$ level has the same subshell configuration as the ground state, and the ionic radius should show no appreciable change. It thus seems reasonable to expect a substantial probability for transitions coupled with low vibrational levels of the complex in its upper state, and so giving sharp lines.

If our other assignments (see below) are correct, the narrow band at $\sim$ 19 000 cm^{-1}, which shows no resolved vibrational structure, is associated with a change of subsheli configuration, while the transitions around 16 000 cm^{-1}, which take place within the same subshell configuration, do show a vibrational structure.

([4]) L. E. Orgel: *Journ. Chem. Phys.*, **23**, 1824 (1955).

([5]) J. Van Vleck: *Journ. Phys. Chem.*, **41**, 67 (1937).

([6]) J. Van Vleck: *Journ. Chem. Phys.*, **8**, 788 (1940).

([7]) K. C. Jørgensen: *Acta Chem. Scand.*, **10**, 887 (1956).

([8]) H. A. Bethe: *Ann. d. Phys.*, **3**, 181 (1929).

It would be interesting to settle the question of the electronic level responsible for the 24 300 cm^{-1} lines by observing the (low-temperature) behaviour of the ion with other ligands than water, so as to increase the spacing between the $^1A_1$ and $^3T_1$ levels with a different field parameter.

Discrimination between $^3A_2 \rightarrow {}^1A_1$ and $^3A_2 \rightarrow {}^3T_1$ for this transition is rather important because if it is the former, then this involves a non-degenerate upper level, which is not split either by non-cubic fields or by spin-orbit coupling, so that the interpretation of the absorption would lead straight to the vibrational behaviour of the complex.

As regards the spacing observed in the vibrational structure, it may be noted that frequency intervals around 175 cm^{-1} have been reported in the optical absorption of rare earth crystals ([9]) and in the Raman spectra of hydrated salts ([10]).

The oscillations of the complex ion are not the only mechanism of energy transfer from the electronic system to the lattice; another process of similar kind could be the excitation of anion oscillations. In our case it is known ([11]) that the SiF_6 group has vibration frequencies of 400, 460 and 650 cm^{-1}. This would give the observed spacing in the observed line patterns; the above considerations about the assignment of electronic levels would still be valid.

One further remark concerning the main absorption band is, that even at the lowest temperatures used, no resolvable splitting was observed, such as might be expected ([12]) to be due to departure from cubic symmetry and consequent lifting of the threefold degeneracy of the $^3T_1$ level.

The Ni^{2+} bands in the 16 000 cm^{-1} region vary from salt to salt. This is consistent with the most probable assignment as an intersystem combination $^3A_2 \rightarrow {}^1E$. In fields of trigonal symmetry ^1E is unsplit, but in lower symmetry (and in tetragonal symmetry) it is split into two.

The band at 19 000 cm^{-1} can be fitted into the theoretical scheme ([2]) as a $^3A_1 \rightarrow {}^1T_1$ transition, in fair agreement with the calculated energy value.

Details of the experimental apparatus and of results on Ni^{2+}, Co^{2+}, Mn^{2+} salts will be given in a following paper.

* * *

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([9]) G. Joos: *Erg. d. Exact. Wiss.*, **18**, 79 (1939).

([10]) B. L. Rao: *Proc. Ind. Ac. Sci.*, A **14**, 41 (1941).

([11]) L. Couture and J. Mathieu: *Journ. Phys. et Rad.*, **12**, 826 (1951).

([12]) J. Owen: *Proc. Roy. Soc.*, A **227**, 183 (1954).

IL NUOVO CIMENTO Vol. VI, N. 2 1° Agosto 1957

Information on the Nature of Heavy Mesons from Photoproduction Experiments.

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(ricevuto il 4 Giugno 1957)

Several analyses of the scattering of K-mesons on nucleons and on nuclei have been made recently by a few authors [1], by means of techniques developed either from field theory and phenomenological models or by means of Montecarlo type calculations. Experimental data are however too poor to give us at present the possibility of determining, by comparison with the theoretical results, some of the quantum numbers there involved. In particular, assuming the K-meson to be of spin 0, the question is still completely open if its parity relative to the hyperons is $+1$ or -1.

As this point is a crucial one for the choice of a suitable K-N-H interaction and therefore for a more refined treatement of K-N scattering, we want here to point out that photoproduction processes of K-mesons (which have been very recently observed [2]) can give us in a rather clear cut way a reliable answer to the problem of relative parity. We shall also notice that other information about K-N interaction can be extracted from the experimental data by using only conservation laws and charge independence hypothesis.

We list here the reactions that have to be expected (on the basis of strangeness conservation) by interaction of γ-rays on hydrogen, together with the relative energy thresholds for the γ's in the Lab. system:

$$a)\quad \gamma+\mathrm{p}\rightarrow \mathrm{K}^{+}+\Lambda^{0} \qquad E_{\gamma}= 930\ \mathrm{MeV}\,,$$

$$b)\quad \rightarrow \mathrm{K}^{0}+\Sigma^{+} \qquad 1\,070\ \mathrm{MeV}\,,$$

$$c)\quad \rightarrow \mathrm{K}+\overline{\mathrm{K}}+\mathrm{N} \qquad 1\,560\ \mathrm{MeV}\,.$$

[1] L. S. Osborne: *Phys. Rev.*, **102**, 290 (1956); G. Cocconi, G. Puppi, G. Quareni and A. Stanghellini: *Nuovo Cimento*, **5**, 172 (1957); C. Ceolin and L. Taffara: *Nuovo Cimento*, **5**, 435 (1957); G. Costa and G. Patergnani: *Nuovo Cimento*, **5**, 448 (1957); D. Amati and B. Vitale: *Nuovo Cimento*, in press; N. Schmitz: private communication; D. Fournet-Davis: private communication.

[2] *Rochester Conference* (1957).

Being the threshold for *c*) considerably higher than the corresponding thresholds for *a*) and *b*), we shall assume in the following that only *a*) and *b*) will take place. This means that only K-mesons will be present as final products without contamination of $\overline{\mathrm{K}}$. Besides, if the energy of the γ beam is very near the corresponding thresholds, no pion will be present in the final states of *a*) and *b*) and therefore we shall have to deal only with two body final states.

We shall make now the assumption (whose validity is of course connected with our previous hypothesis that the energy of the γ beam is very close to the threshold value so that final states have small kinetic energy in the C.M. system) that only S and P waves are present in the relative K-H motion. This will limit sharply the number of multipoles that can contribute to *a*) and *b*). We give in the following table a list of all the initial and final states allowed by total momentum and parity conservation both for « scalar » K-mesons (this means: relative parity K-H $= +1$) and « pseudoscalar » K-mesons (relative parity $= -1$):

Multipole	J	« scalar » K	« pseudoscalar » K
$M1$	$\frac{1}{2}$	$S(\frac{1}{2})$	$P(\frac{1}{2})$
	$\frac{3}{2}$	—	$P(\frac{3}{2})$
$E1$	$\frac{1}{2}$	$P(\frac{1}{2})$	$S(\frac{1}{2})$
	$\frac{3}{2}$	$P(\frac{3}{2})$	—
$E2$	$\frac{3}{2}$	—	$P(\frac{3}{2})$
	$\frac{5}{2}$	—	—
$M2$	$\frac{3}{2}$	$P(\frac{3}{2})$	—
	$\frac{5}{2}$	—	—

We consider now the angular distribution of the final products in the C.M. system. In both the « scalar » and the « pseudoscalar » cases S and P wave K-mesons will be photoproduced and interferences have to be expected among the different allowed final states. We note however that if we limit ourselves to only neutral K-mesons, K^0, the situation is considerably simplified. Neutral K-mesons as a matter of fact cannot be produced by « photoelectric » or by « catastrophic » transitions (we refer here to the classification given by MARSHAK [3]) but only by « shaking-off » transitions. In other words, they have to be emitted directly by the nucleon and cannot change orbital angular momentum by direct interaction with the electromagnetic field. This means that « scalar » K^0 will be produced only in a S state (if we disregard, as it seems reasonable at threshold, contributions coming from the recoil of the nucleons) and « pseudoscalar » K^0 only in a P state.

This gives us two possibilities for finding out from experiment the sign of the relative K-H parity. First, we can analyse the momentum dependence of the cross section near threshold, where the cross section should vary as q in the « scalar » case and as q^3 in the « pseudoscalar » case. Besides, a rather more direct investigation could be made on the angular distributions of the produced K^0. Disregarding the electric quadrupole contribution to the $P(\frac{3}{2})$ « pseudoscalar » case for simplicity (its inclusion would not change the qualitative results that follow) we obtain the following angular distributions for the two cases:

« scalar » K^0:

$$w(\vartheta) = |a_{M1(\frac{1}{2})}|^2 ,$$

« pseudoscalar » K^0:

$$w(\vartheta) = |a_{M1(\frac{1}{2})}|^2 + |a_{M1(\frac{3}{2})}|^2 \,(1 + \tfrac{3}{2}\sin^2\vartheta) + \\ + \mathrm{Re}\,(a_{M1(\frac{1}{2})} a_{M1(\frac{3}{2})})\,(3\cos\cos^2\vartheta - 1) = \\ = A + B\cos^2\vartheta ,$$

where the a's represent the scattering amplitudes for the corresponding dipole transitions. We see that the « scalar » K^0 must be isotropic in the C.M. system and

[3] R. E. MARSHAK: *Mesons Physics* (New York, 1952) p. 6.

the « pseudoscalar » K^0 must follow a $A + B\cos^2\vartheta$ distribution. The relative sign of A and B cannot be foreseen only from general principles but has to be obtained by a more detailed model of the interaction.

We want now to say a few words more about the information that we could get from photoproduction of K-mesons on the K-Σ interaction. Final states will be a mixture of isobaric spin $T = \frac{1}{2}$ and $T = \frac{3}{2}$ eigenstates and we can easily define scattering amplitudes $a_{\frac{1}{2}}$ and $a_{\frac{3}{2}}$ for the different dipole transitions involved. We can therefore analyse the experimental data in terms of final isobaric spin eigenstates and from this deduce informations about the relative weight of the $T = \frac{1}{2}$ and $T = \frac{3}{2}$ K-Σ eigenstates. On the other hand this analysis will also allow us to connect the phases of the a's with the K-H scattering phase shifts in the way already indicated by Watson for the case of photoproduction of pions [4].

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[4] K. M. Watson: *Phys. Rev.*, **95**, 228 (1954).

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A Note Concerning the Electric Hexadecapole Moment of the First Excited ^{111}Cd Nucleus.

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(ricevuto il 4 Giugno 1957)

It is well known that one of the important experimental evidences of extranuclear effects on the γ-γ angular correlation was found by the investigation of successive γ-rays emitted from ^{111}Cd which follow the electron capture decay of ^{111}In. Among these successive researches an interesting experiment of the Zürich group [1] in regard to a new multipole moment of the excited state of the nucleus was performed by making use of a face-centered cubic crystal of Ag, in which the excited ^{111}In nuclei were homogeneously embedded.

In this case it was expected that the electric quadrupole interaction should vanish because of the symmetry of extra nuclear fields, and consequently the interaction of a higher multipole moment, i.e. the electric hexadecapole moment, with the third derivative of crystalline fields should come in evidence.

For extranuclear cubic fields the attenuation factor $A_{\max}/A_{\min}$ of the anisotropy $A = W(\pi)/W(\pi/2) - 1$ (< 0) defined by two definite correlation functions $W(\theta)$ with $\theta = \pi$ and $\pi/2$ was derived as follows:

$$\frac{A_{\max}}{A_{\min}} = \frac{1}{1 + \frac{1}{(a\omega\tau_N)^2}}, \tag{1}$$

with a numerical factor a, based upon our theoretical foundation for the empirical formula proposed by the Zürich group, where ω is the coupling parameter of the nuclear moment with external field and τ_N means the lifetime of the intermediate nuclear level (excited by 247 keV and having spin $\frac{5}{2}\hbar$ and even parity) involved in this γ-cascade.

The experimental value of the attenuation factor obtained by the above group was $(1.032)^{-1}$, then the coupling parameter was given by

$$\omega \sim 3.3 \cdot 10^7 \text{ Hz}, \tag{2}$$

where they used $\tau_N = 1.25 \cdot 10^{-7}$ s, and a in our formula has the value of 24^{-1} for the electric hexadecapole interaction. If their value (in the position of the nucleus in question) of the third derivative $\mathrm{d}^3E_z/\mathrm{d}z^3$ of the crystalline field estimated by making use of a simple model of the crystal structure could be given by the order of magnitude of

[1] E. Heer and R. Rüetschi: *Helv. Phys Acta*, **28**, 525 (1955).

10^{38} $e\cdot\mathrm{cm}^{-5}$, the value of the electric hexadecapole moment of the excited state of ^{111}Cd should become 10^{-39} $e\cdot\mathrm{cm}^4$.

On one hand, however, the value of the above nuclear moment calculated by the theory of nuclear shell model was the order of magnitude of 10^{-51} $e\cdot\mathrm{cm}^4$. Therefore we think that this apparent discrepancy is too large to regard their interpretation to be correct, although there exist some ambiguities in the estimation of the extranuclear parameters.

Now in the first place it is expected for one of the possible considerations about these results that the order of magnitude of the nuclear electric hexadecapole moment calculated by the collective model may become larger in general than that by the individual particle model. But the former gave the value of the order of magnitude of 10^{-48} $e\cdot\mathrm{cm}^4$, which is still much smaller than the value given by angular correlation experiments. We note here that the value of the electric hexadecapole moment of the ground state of ^{121}Sb ($d_{\frac{5}{2}}$, odd proton) or ^{123}Sb ($g_{\frac{7}{2}}$, odd proton) was estimated as 10^{-51} $e\cdot\mathrm{cm}^4$ from pure quadrupole resonance experiments [2], whose value is not so different from the value calculated by the theory of a collective model [3].

Thus it is suggestive that the above value of 10^{-48} $e\cdot\mathrm{cm}^4$ of the present moment is an upper bound by using the simple collective model, and then we suppose that it is difficult to expect still larger values than this order of magnitude. In the same way even removing the uncertanty on the value of the third derivative of the crystalline field may be impossible to cover this large discrepancy.

On the other hand, we should examine the states of extranuclear fields. As their experiments were performed at room temperature, a possibility is expected that the lattice vibration belonging to a normal mode of deformation may break the cubic symmetry of the crystalline fields. So we can expect the effect of an interaction between the nuclear electric quadrupole moment of the intermediate state and an electrostatic field gradient induced by phonons on the angular correlation of the γ-cascade. This quadrupole interaction at room temperature was proved to lead to a value of the coupling parameter:

$$|\omega| \sim 1.8\cdot 10^6 \text{ Hz}\,, \tag{3}$$

in good agreement with the order of magnitude of their experimental value:

$$\omega \sim 6.8\cdot 10^6 \text{ Hz}\,, \tag{4}$$

where in this case the numerical factor a in our formula (1) is equal to 5^{-1}, and the value of the electric quadrupole moment of the excited ^{111}Cd nucleus was assumed as $3.0\cdot 10^{-25}$ $e\cdot\mathrm{cm}^2$, which was obtained from the theory of shell structures [4].

In conclusion, it can be considered that the above angular correlation experiment could not find the effect of the electric hexadecapole moment of the first excited state of the ^{111}Cd nucleus, but could be interpreted by the electric quadrupole interaction induced by lattice phonons. It will be possible to expect an extranuclear effect of the pure hexadecapole moment in question, when the experiment will be performed with higher accuracy at low temperature.

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[2] T. C. Wang: *Phys. Rev.*, **99**, 566 (1955).

[3] Y. Yamaguchi: private communication; S. Suekane and Y. Yamaguchi: *Progr. Theor. Phys.*, **17**, 449 (1957).

[4] The value obtained from angular correlation experiments was $2.6\cdot 10^{-25}$ $e\cdot\mathrm{cm}^2$; H. Albers-Schönberg, E. Heer, T. B. Novey and P. Scherrer: *Phys. Rev.*, **91**, 1287 (1953); *Helv. Phys. Acta*, **27**, 545 (1954).

A Gauge Invariant Lagrangian for a Non-Local Electrodynamics.

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(ricevuto il 7 Giugno 1957)

We should like to derive a gauge and Lorentz invariant Lagrangian for a type of the convergent quantum electrodynamics (1) which preserves the structure of the original Dirac Maxwell equations, and shall find it possible provided the gauge transformations are also accordingly modified.

The variational derivative of the Lagrangian

$$(1) \qquad \mathcal{L}\left(\overline{\psi}''(1+2b), \ldots \psi''(1+2b), \frac{\partial}{\partial 1_\mu}\psi''(1+2b), \psi''(1), \ldots A''_\mu(1+2b), \ldots\right),$$

with respect to the field ψ is

$$(2) \qquad \frac{\delta\mathcal{L}}{\delta\psi} = \frac{\delta\mathcal{L}}{\delta\psi(1+2b)} + \frac{\delta\mathcal{L}}{\delta\psi(1)} + \ldots,$$

and

$$(3) \qquad \frac{\partial\psi(1+\alpha b)}{\partial\psi(1+\beta b)} = \delta_{\alpha\beta}\,.$$

Then the application of the variational principle to the Lagrangian (2)

$$(4) \qquad \mathcal{L} = -\frac{1}{2}\left(\frac{\partial}{\partial 1_\nu}A''_\mu(1,0)\right)\left(\frac{\partial}{\partial 1_\nu}A''_\mu(1,0)\right) -$$

$$-\frac{e}{4\lambda}\left\{\overline{\psi}''(1,-2b)\left(H(1)-i\lambda a''(1,0)\right)\psi''(1,0) + \overline{\psi}''(1,0)\left(H(1)-i\lambda a''(1,2b)\right)\psi''(1,2b) + \right.$$

$$\left. + \overline{\psi}''^{c}(1,-2b)\left(H(1)+i\lambda a''(1,0)\right)\psi''^{c}(1,0) + \overline{\psi}''^{c}(1,0)\left(H(1)+i\lambda a''(1,2b)\right)\psi''^{c}(1,2b)\right\},$$

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(1) P. A. M. Dirac: *Quantum Mechanics*. Third Edition (Oxford, 1947); P. Sen: *Nuovo Cimento*, **3**, 390 (1956). The notation and procedure of this paper are followed here.

(2) J. Schwinger: *Phys. Rev.*, **74**, 1439 (1948).

where, for instance, the fields $\psi''(1, 2b)$ and $\psi''(1, 0)$ are defined by

$$2\psi''(1, 2b) = \psi''(1 + 2b) + \psi''(1 - 2b), \qquad \psi''(1, 0) = \psi''(1), \tag{5}$$

gives the wave equations

$$\begin{cases} (H(1) - i\lambda a''(1, b))\psi''(1, b) = 0, & \bar{\psi}''(1, -b)(\bar{H}(1) + i\lambda a''(1, b)) = 0, \\ (H(1) + i\lambda a''(1, b))\psi''^{c}(1, b) = 0, & \bar{\psi}''^{c}(1, -b)(\bar{H}(1) - i\lambda a''(1, b)) = 0, \end{cases} \tag{6}$$

$$\Box^2 A''_\mu(1, 0) = \frac{ie}{4}\{\bar{\psi}''(1, 2b)\gamma_\mu\psi''(1, 0) + \bar{\psi}''(1, 0)\gamma_\mu\psi''(1, 2b) + \tag{7}$$

$$+ \bar{\psi}''^{c}(1, 2b)\gamma_\mu\psi''^{c}(1, 0) + \bar{\psi}''^{c}(1, 0)\gamma_\mu\psi''^{c}(1, 2b)\} = -\frac{1}{c}j''_\mu(1).$$

Here b_e and b_p have been put equal to b for convenience. We note that

$$j''_\mu(1) = \frac{\partial\mathcal{L}}{\partial(\partial\psi''/\partial 1_\mu)} - \frac{\partial\mathcal{L}}{\partial(\partial\bar{\psi}''^{c}/\partial 1_\mu)}. \tag{8}$$

To solve the equations (6) we define the Feynman general electron propagation operators $K''(1, 2, b)$ and $\overset{c}{K}{}''(1, 2, b)$ by the relations

$$\begin{cases} (H(1) - i\lambda a''(1, b))K''(1, 2, b) = i\,\delta(12), \\ \qquad\qquad K''(1, 2, b)(\bar{H}(2) + i\lambda a''(2, b)) = -i\,\delta(12), \\ (H(1) + i\lambda a''(1, b))\overset{c}{K}{}''(1, 2, b) = -i\,\delta(12), \\ \qquad\qquad \overset{c}{K}{}''(1, 2, b)(\bar{H}(2) - i\lambda a''(2, b)) = i\,\delta(12), \end{cases} \tag{9}$$

and

$$2K''(1, 2, b) = K''(1 + b,\ 2 + b) + K''(1 - b,\ 2 - b). \tag{10}$$

We note that $L''(1, 2, b)$ is differently defined [1]. Then

$$\begin{cases} K''(1, 2, b) = K(12) + \lambda\int K(13)a(3, b)K''(3, 2, b)\mathrm{d}3, \\ \overset{c}{K}{}''(1, 2, b) = \overset{c}{K}(12) + \lambda\int \overset{c}{K}(13)a(3, b)\overset{c}{K}{}''(3, 2, b)\mathrm{d}3. \end{cases} \tag{11}$$

Furthermore

$$j''_\mu(1) = -\frac{iec}{4}\,\mathrm{Tr}\,\gamma_\mu\,\{K''(1 + b,\ 1 - b,\ b) + K''(1 - b,\ 1 + b,\ b) + \tag{12}$$

$$+\overset{c}{K}{}''(1 + b,\ 1 - b,\ b) + \overset{c}{K}{}''(1 - b,\ 1 + b,\ b)\} = -iec\lambda\int \Pi''_{\mu\alpha}(1, 2, b)A''_\alpha(2, 0)\mathrm{d}2,$$

where

$$2\,\Pi''_{\mu\alpha}(1, 2, b) = \tag{13}$$

$$= \mathrm{Tr}\,\gamma_\mu\,\{\overset{c}{K}{}''(1 + b,\ 2,\ b)\gamma_\alpha K''(2,\ 1 - b,\ b) + \overset{c}{K}{}''(1 - b,\ 2,\ b)\gamma_\alpha K''(2,\ 1 + b,\ b)\},$$

and we obtain the photon propagation equations

$$\Box^2(1)\, 2L''_{\mu\nu}(1,2) = i\hbar c\, \delta_{\mu\nu}\, \delta(12) + i\hbar c\lambda^2 \int \Pi''_{\mu\alpha}(1,3,b) 2L''_{\alpha\nu}(3,2)\mathrm{d}3\,, \tag{14}$$

$$L''_{\mu\nu}(1,2) = \frac{1}{2} L_\mu(12) + \lambda^2 \iint L_{\mu\alpha}(13)\, \Pi''_{\alpha\beta}(3,4,b) L''_{\beta\nu}(4,2)\mathrm{d}3\mathrm{d}4\,. \tag{15}$$

Finally we note that the Lagrangian (4) is invariant under the gauge transformations

$$\left\{\begin{array}{l} A''_\mu(1,2b) \to A''_\mu(1,2b) - \dfrac{\partial}{\partial 1_\mu} \Lambda(1,b)\,, \\ \qquad\qquad\qquad\qquad A''_\mu(1,0) \to A''_\mu(1,0) - \dfrac{\partial}{\partial 1_\mu} \Lambda(1,b)\,, \\ \psi''(1,2b) \to \exp\left[-i\lambda\Lambda(1,b)\right]\psi''(1,2b)\,, \\ \qquad\qquad\qquad\qquad \psi''^c(1,2b) \to \exp\left[i\lambda\Lambda(1,b)\right]\psi''^c(1,2b)\,, \end{array}\right. \tag{16}$$

where

$$2\Lambda(1,b) = \Lambda(1+b) + \Lambda(1-b)\,, \tag{17}$$

provided

$$\Box^2(1)\Lambda(1,0) = 0\,, \tag{18}$$

and that charge conservation can be derived from its gauge invariance.

On Heavy Primary Cascades.

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(ricevuto il 16 Giugno 1957)

In recent months, several cases have been reported [1-3] of the observation in nuclear emulsions of multiple disintegrations produced by heavy primary cosmic ray nuclei. These events have been distinguished by having annomalously short mean free paths, and it has been suggested that they provide evidence for some abnormality of these nuclei. Indeed it has been considered [2] that these events suggest the presence of anti-matter in the primary cosmic radiation. It is the purpose of this letter to show that the observation of these events is not significant, if the finite size of the detecting emulsions is taken into account.

The probability of observing a series of interactions in an infinitely large detector is given by:

$$P_\infty = \prod_i \left\{1 - \exp\left[-x_i/\lambda_i\right]\right\},$$

where x_i is the distance traversed by an i-type particle having a mean free path of λ_i. The probability of observing a similar series of interactions in a detector of finite size is given by:

$$P_{\text{finite}} = \prod_i \frac{\left\{1 - \exp\left[-x_i/\lambda_i\right]\right\}}{\left\{1 - \exp\left[-x_0/\lambda_i\right]\right\}},$$

where x_0 is the distance over which an interaction could have been observed.

Thus, in the first event reported by Tokunaga *et al.* [3], P_∞ equals $2 \cdot 10^{-4}$ but P_{finite} is only $1.8 \cdot 10^{-2}$, and the probability of observing such an event in a stack of 48 600 μm 10×15 cm emulsions is appreciable. Similarly, the event reported by Milone [1] has a P_∞ of $1 \cdot 10^{-3}$ but a P_{finite} of only $4.1 \cdot 10^{-2}$ and again the probability of observing the event is appreciable. A similar arguement presumably applies to the event reported by Yagoda [2], although the details are not available.

It would appear, therefore, that these events do not represent any abnormal behaviour of the heavy nuclei in the cosmic radiation, but have been observed because of the highly biased sample of events recorded by a detector of limited dimensions.

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[1] A. Milone: *Suppl. Nuovo Cimento*, **12**, 354 (1954).

[2] H. Yagoda: *Bull. Am. Phys. Soc.*, Series II, 64 (1956).

[3] S. Tokunaga, T. Ishii and K. Nishikawa: *Nuovo Cimento*, **5**, 517 (1957).

Multiple Emission in Nuclear Reactions.

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(ricevuto il 18 Giugno 1957)

An excited nucleus is generally supposed to go to the ground state through a succession of elementary processes in which only one particle at a time is emitted. While this is quite clear at low excitation energy, we can think that at higher energy two or more particles can be emitted at the same time, in such a way that they will result correlated. Here we will consider only heavy nuclei, in which we shall suppose that a statistical theory is valid.

Let us consider a nucleus C, to which has been given an excitation energy E_c; we take the probability of emission of n particles, with total kinetic energy between ε and $\varepsilon + \mathrm{d}\varepsilon$, proportional to:

$$g_{12\ldots n}|H_{0f}|^2 \varrho_n V^n \omega(E_f) ,$$

where: $E_f = E_c - \varepsilon - B$; $B = \sum_1^n B_i$ is the total separation energy of the n particles; $g_{12\ldots n}$ is the weight of the states of spin of the n particles; H_{0f} is the transition matrix element; $\varrho_n V^n$ is the number of states in the phase space; $\omega(E_f)$ is the level density of the final nucleus. In the non-relativistic approximation is

$$\varrho_n = \frac{(m_1 m_2 \ldots m_n)^{\frac{3}{2}}}{2^{\frac{3}{2}n}\pi^{\frac{3}{2}n}\hbar^n} \frac{\varepsilon^{\frac{3}{2}n-1}}{(\frac{3}{2}n-1)!} ,$$

where $m_1, m_2, \ldots, m_n$ are the masses of the particles. Now we suppose that H_{0f} is simply proportional to the probability that the particles are at the same time inside the nuclear volume Ω; if the particles are uncharged we can take this probability equal to $(\Omega/V)^n$. Therefore the probability of emission of n particles will result:

$$(1) \qquad W_n(\varepsilon)\,\mathrm{d}\varepsilon = K g_{12\ldots n} \frac{\Omega^n (m_1 m_2 \ldots m_n)^{\frac{3}{2}}}{(2\pi)^{\frac{3}{2}n}\hbar^{3n}} \frac{\varepsilon^{\frac{3}{2}n-1}}{(\frac{3}{2}n-1)!}\, \omega(E_c - \varepsilon - B)\,\mathrm{d}\varepsilon ,$$

(1) V. Weisskopf: *Phys. Rev.*, **52**, 295 (1937).

where K is a constant. We put

$$(\Omega = \frac{4\pi}{3}\left(\frac{h}{\mu}\right)^3 A ,$$

(μ mass of the π-meson); therefore

$$W_1(\varepsilon)\,\mathrm{d}\varepsilon = Kg_1 \frac{4A}{3\sqrt{2}\pi} \frac{m_1^{\frac{3}{2}}}{\mu^3} \varepsilon^{\frac{1}{2}} \omega(E_c - \varepsilon - B_1)\,\mathrm{d}\varepsilon , \tag{2}$$

$$W_2(\varepsilon)\,\mathrm{d}\varepsilon = Kg_{12} \frac{A^2}{9\pi} \frac{(m_1 m_2)^{\frac{3}{2}}}{\mu^6} \varepsilon^2 \omega(E_c - \varepsilon - B_1 - B_2)\,\mathrm{d}\varepsilon ,$$

$$W_3(\varepsilon)\,\mathrm{d}\varepsilon = Kg_{123} \frac{32\sqrt{2}A^3}{2\,835\pi^2} \frac{(m_1 m_2 m_3)^{\frac{3}{2}}}{\mu^9} \varepsilon^{\frac{7}{2}} \omega(E_c - \varepsilon - B_1 - B_2 - B_3)\,\mathrm{d}\varepsilon . \tag{4}$$

The total rate of emission is

$$W_{\mathrm{tot}} = \int_0^{E-B} W(\varepsilon)\,\mathrm{d}\varepsilon .$$

At high excitation energy the emission of nucleons or of nuclear complexes is the unique process by which a nucleus looses energy; so we are not interested in the calculation of K, or of the lifetime, but only in the calculation of the relative probability of emission of the various types of particles.

If we take $\omega(E) = b \exp[2\sqrt{aE}]$ (b and a are some constants) and consider a nucleus $A = 100$, $a = 6\ \mathrm{MeV}^{-1}$, we obtain the ratios $R_2 = W_{2\mathrm{tot}}/W_{1\mathrm{tot}}$ and $R_3 = W_{3\mathrm{tot}}/W_{1\mathrm{tot}}$ given in Table I

TABLE I.

E_c (MeV)	R_2	R_3
50	0.06	0.00
100	0.24	0.08
150	0.54	0.19
180	0.80	0.26

For a charged particle the probability of emission is obtained multiplying (2) by the transmission coefficient of the Coulomb barrier $T(\varepsilon)$. Let us calculate the probability of emission of two particles, one of which is charged. At any ε the energy spectrum of an uncharged particle is

$$n(\varepsilon_1) = \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)} ;$$

if the particle is charged the spectrum is

$$n_c(\varepsilon_1) = \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)}\,T(\varepsilon_1) .$$

We take

$$T(\varepsilon_1) = 0 \qquad \text{if} \qquad \varepsilon_1 < V_1 ,$$

$$T(\varepsilon_1) = 1 \qquad \text{if} \qquad \varepsilon_1 > V_1 ,$$

where V_1 is the reduced height of the barrier; in nuclei $A = 100$ V_1 is ~ 7 MeV for protons and ~ 15 MeV for α particles. In this case (3) must be multiplied by the factor

$$G_1(\varepsilon) = \frac{\int_{V_1}^{\varepsilon} \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)}\, \mathrm{d}\varepsilon_1}{\int_{0}^{\varepsilon} \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)}\, \mathrm{d}\varepsilon_1} = 1 - \frac{2}{\pi} \sin^{-1} \eta^{\frac{1}{2}} + \frac{2}{\pi} [\eta(1-\eta)]^{\frac{1}{2}} (1 - 2\eta) ,$$

where $\eta = V_1/\varepsilon$. If both the particles are charged, the particle 1 is emitted only if $V_1 < \varepsilon < \varepsilon - V_2$. In this case (3) must be multiplied by

$$G_2(\varepsilon) = \frac{\int_{V_1}^{\varepsilon - V_2} \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)}\, \mathrm{d}\varepsilon_1}{\int_{\varepsilon}^{} \sqrt{\varepsilon_1(\varepsilon - \varepsilon_1)}\, \mathrm{d}\varepsilon_1} = \frac{2}{\pi} [\sin^{-1}(1 - \eta_2) - \sin^{-1} \eta_1] +$$

$$+ \frac{2}{\pi} \left\{ [\eta_1(1 - \eta_2)]^{\frac{1}{2}} (1 - 2\eta_1) + [\eta_2(1 - \eta_2)]^{\frac{1}{2}} (1 - 2\eta_2) \right\} .$$

Some results are given in Tables II, III, IV.

TABLE II. – *Relative probabilities of emission for two energies of excitation.*

E_c	W_{1n}	W_{1p}	$W_{1\alpha}$	W_{2n}	W_{np}	$W_{n\alpha}$	W_{2p}	$W_{2\alpha}$	W_{3n}
100 MeV	46%	14%	14%	11%	5%	4%	1%	1%	4%
160 MeV	22%	9%	14%	14%	10%	15%	4%	8%	4%

TABLE III. – *Ratio R_n between the total probability that a neutron is emitted with other particles and the probability that it is emitted alone.*

E_c	50	100	150	180
R_n	0.1	0.7	1.8	2.8

We obtain a grater probability of neutron emission, and a grater ratio N_α/N_p than those obtained with the Weisskopf formula. Moreover in our case the particles are emitted with energy lower than in the case of the Weisskopf formula: therefore the number of neutrons emitted in a complete spallation is still increased. We remark that these results can improve the situation in the problem of the excitation of nuclei by absorption of π^--mesons at rest (2).

(2) V. DE SABBATA, E. MANARESI and G. PUPPI: *Nuovo Cimento*, **10**, 1704 (1953); A. TOMASINI: *Nuovo Cimento*, **3**, 160 (1956).

TABLE IV. – *Relative probabilities of emission of neutrons, protons and particles (by any process) compared with the results of the Weisskopf formula* [1].

	$E_c = 100$ MeV		$E_c = 160$ MeV	
	calculated	W. formula	calculated	W. formula
N_n	70%	45%	54 %	37%
N_p	15%	35%	17.5%	33%
N_α	15%	20%	28.5%	30%
N_α/N_p	1.0	0.54	1.63	0.91

The particles emitted simultaneously must show a strong angular correlation. The correct energy balance between the initial and the final state is:

$$E_f = E_c - \Lambda - \varepsilon - \varepsilon_N ,$$

where ε_N is the kinetic energy of the residual nucleus. ε_N is negligible in the calculation of the probabilities of emission, but not in the calculation of the angular correlations. Let us consider the case of emission of two particles; if the two particles are emitted in the same direction ε_N is maximum and E_f minimum, while if they are emitted in opposite directions E_f is maximum. Since $\omega(E_f)$ is a function strongly increasing with E_f, the two particles are emitted preferentially in opposite directions. For instance for two particles of the same mass the correlation is approximately of the type:

$$\exp\left[2\frac{m}{M}\right]\sqrt{a\varepsilon_1\varepsilon_2}\cos\theta , \tag{5}$$

where M is the mass of the residual nucleus. In the case of emission of two α-particles the ratio between the probability of emission at 180° and the probability of emission at 0° is $\sim 10^4$, taking $E_c = 100$ MeV, and $\varepsilon_1 = \varepsilon_2 = 10$ MeV (mean energy of emission). (5) shows also an energy correlation, weaker than the angular one.

LIBRI RICEVUTI E RECENSIONI

C. L. SIEGEL – *Vorlesungen über Himmelsmechanik*. Ed. Springer, Berlin - Göttingen - Heidelberg, 1956.

Questo Corso di Meccanica Celeste dell'insigne matematico C. L. SIEGEL di Göttingen costituisce una sintesi di conferenze e corsi tenuti da lui a Francoforte sul Meno, Baltimore, Göttingen e Princeton. Il libro non va considerato un trattato per la determinazione pratica ad esempio delle orbite di pianeti, ma una esposizione di più o meno recenti ricerche sul problema degli n corpi, con un criterio di scelta dettato dai gusti personali dell'Autore, a cui si devono anche non pochi contributi originali in materia. L'interesse che l'opera stessa può suscitare in un fisico o in un ingegnere, è da cercarsi non tanto nei risultati concernenti l'Astronomia, — piuttosto esigui nel terzo Capitolo, pur molto interessante, dedicato a problemi di stabilità, — ma nel trovar riuniti, in una presentazione concisa e suggestiva, vari metodi per la trattazione « in grande » di equazioni differenziali, utili anche in altre ricerche.

Il libro è suddiviso in tre Capitoli, il contenuto dei quali passerò in rapida rassegna.

Cap. I. – *Il Problema dei tre Corpi.*

Dopo alcuni paragrafi di carattere preliminare — sulla covarianza delle derivate di Lagrange, sulle trasformazioni canoniche (che conservano ad un sistema di equazioni canoniche la proprietà di essere, appunto, un sistema di equazioni canoniche) e sull'equazione a derivate parziali di Hamilton e Jacobi —, l'Autore passa all'esposizione del problema degli n corpi. Tale problema consiste, come è noto, nella descrizione per tutti i valori del tempo dell'andamento di tutte le soluzioni delle equazioni del moto attinenti alla funzione potenziale della legge di Newton per condizioni iniziali qualsiansi, cioè delle equazioni

$$m\ddot{q} = U_q \quad \text{con} \quad U = \sum_{k>l} \frac{m_k m_l}{r_{kl}},$$

ove m_k e m_l sono rispettivamente la massa del k-esimo e dello l-esimo corpo e r_{kl} la loro distanza. Vengono elencati i dieci classici integrali del problema degli n corpi (integrali del baricentro, delle aree e dell'energia), e si accenna ad un risultato di H. BRUNS, secondo il quale non possono esistere altri integrali algebrici. Dopo aver escluso la collisione contemporanea di tutti gli n corpi (possibile soltanto se tutte le costanti delle aree sono nulle) viene dimostrato il teorema di A. Sundman sulla regolarizzazione del problema dei tre corpi. Si può infatti scegliere la variabile indipendente in modo tale che le equazioni differenziali del moto rimangono regolari anche attraverso una collisione fra due dei tre corpi. Da matematico, l'Autore non si preoccupa della « utilità » di questa regolarizzazione — cioè della questione se le soluzioni prolungate oltre la collisione possono descrivere ciò che dopo una simile catastrofe potrà in realtà accadere —, ma chiude il primo Capitolo con le seguenti parole (che riassumono anche due Lemmi di Sundman):

« È possibile calcolare due numeri positivi ϱ e ε tali che in una eventuale collisione fra Terra e Sole la Luna abbia dalla Terra almeno la distanza ϱ, e occorra almeno il tempo ε, prima che la Luna, dal suo canto, collida con la Terra. Tale risultato della Teoria di Sundman ci aiuterà a guardare all'avvenire con più fiducia ».

Cap. II. – *Soluzioni periodiche.*

Dato che i risultati di Sundman non possono essere estesi al problema di più di tre corpi, l'Autore passa a determinare un tipo di soluzioni particolari specialmente interessante, cioè le soluzioni periodiche. Come si vede dalle equazioni del moto, non possono esistere soluzioni del problema che non dipendano dal tempo, soluzioni che sarebbero eo ipso periodiche e verrebbero chiamate soluzioni d'equilibrio. Tuttavia la ben nota soluzione di Lagrange (nella quale i tre punti materiali si muovono su orbite circolari uniformemente in un piano fisso) può essere interpretata come una soluzione d'equilibrio riferita ad un sistema di coordinate ruotante. Si dà un teorema secondo cui esistono soluzioni periodiche in vicinanza della soluzione di Lagrange, sotto ipotesi che si esprimono in termini degli autovalori del sistema di equazioni differenziali che si ottiene quando si conservino nel sistema Hamiltoniano soltanto i termini lineari. L'Autore espone poi il ben noto problema ristretto dei tre corpi e particolarmente il problema lunare di Hill, dei quali si conoscono soluzioni periodiche. Segue una generalizzazione del problema di Hill, la quale fornisce soluzioni periodiche dell'esatto problema dei tre corpi, soluzioni che hanno come caso limite quelle di Hill. Infine l'Autore espone due metodi generali per la determinazione di soluzioni periodiche: il metodo di continuità e il metodo dei punti fissi, i quali tutti e due risalgono a Poincaré, ma sono stati sviluppati e perfezionati da Birkhoff, dal cui libro sui sistemi dinamici [1] l'Autore ha tratto vari suggerimenti.

Cap. III. – *Problemi di Stabilità.*

Considerando in un primo tempo trasformazioni di uno spazio topologico generale in vicinanza di un punto fisso a della trasformazione stessa, l'Autore distingue trasformazioni non-instabili e trasformazioni instabili, le prime caratterizzate dal fatto che ogni intorno del punto fisso contiene un insieme invariante costituito non soltanto dal punto fisso a stesso. Fra le trasformazioni non-instabili figurano quelle stabili per le quali ogni intorno del punto fisso contiene anzi un intorno fisso. Queste definizioni si traducono in analoghe sulla stabilità delle soluzioni d'equilibrio di un sistema di equazioni differenziali e in particolare di un sistema Hamiltoniano. L'Autore dimostra poi i noti teoremi di Ljapunow e di Dirichlet concernenti la stabilità di un sistema di equazioni differenziali. I risultati che se ne derivano per la risoluzione del problema di stabilità dei sistemi Hamiltoniani (e perciò anche per il problema degli n corpi) sono un po' frammentari — come ammette lo stesso Autore —, ma dànno vari suggerimenti per ulteriori ricerche. La definizione di stabilità viene estesa anche alle soluzioni periodiche di un sistema Hamiltoniano. In ciò vengono utilizzate trasformazioni che conservano le aree (inhaltstreue Abbildungen) e vengono riportati risultati di Levi-Civita e di Fermi. Nell'ultimo paragrafo l'Autore porta il Teorema del ritorno di Poincaré e ne trae una applicazione al problema degli n corpi.

Il libro è corredato di una bibliografia che non vuole essere considerata esauriente, riferendosi l'Autore ad una bibliografia più completa, contenuta in un

[1] G. D. Birkhoff: *Dynamical Systems*, American Mathematical Society, Colloquium Publications, vol. IX (New York, 1927).

libro di A. WINTNER [2]. Pare non superfluo accennare a qualche contributo più recente italiano al problema in questione, come alcuni lavori di C. AGOSTINELLI (fra i quali due espositivi [3]) e di G. ARMELLINI [4].

La veste tipografica è quella solita accurata della collana « Die Grundlagen der Mathematischen Wissenschaften in Einzeldarstellungen » della casa editrice Springer, collana di cui l'opera costituisce il Volume LXXXV. Un rilievo tipografico dei risultati più salienti avrebbe però giovato a facilitarne la lettura.

M. J. DE SCHWARZ

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[2] A. WINTNER: *The Analytical Foundations of Celestial Mechanics* (Princeton, 1947).

[3] C. AGOSTINELLI: *Sul problema dei tre corpi*, in *Matematiche*, **5**, 45 (1950) e *Rend. Sem. Mat. Fis.*, Milano, **21**, 165 (1951)

[4] G. ARMELLINI: *Sul problema lunare di Hill*, in *Atti Accad. Naz. Lincei, Rend. Cl. Sci. Fis. Mat. Nat.* (8), **4**, 352 (1948).

Journal of the Institution of Telecomunication Engineers. Special Five Year Plan Number, Giugno-Settembre 1956. (The Institution of Telecommunication Engineers, New Delhi).

Un organico programma di industrializzazione si va attuando in India da quando questo Paese ha ottenuto l'indipendenza; contemporaneamente e con ritmo anche più rapido si sviluppa la rete di telecomunicazioni, senza la quale ogni forma di vita moderna risulterebbe paralizzata. Il volume che segnaliamo, della rivista indiana *Journal I.T.E.*, offre una documentazione dei progressi compiuti nel campo delle telecomunicazioni durante il primo piano quinquennale 1951-56.

Le linee telegrafiche e telefoniche, limitate precedentemente alle aree delle grandi città, si diramano ora nei centri minori e talvolta fin nei villaggi. Il numero di abbonati che nel 1951 era 168400 passa a 265500 con un incremento del 60%. Il traffico telefonico, valutato in numero di chiamate effettuate in un anno, risulta quasi triplicato. Intanto si installano stazioni telescriventi e si procede alla posa di cavi sotto traccia prima totalmente inesistenti.

Nel 1947, anno della riacquistata libertà, l'area coperta dalle trasmissioni radio con una intensità di campo di 1 mV/m era di circa 44000 miglia quadrate e serviva una popolazione di 32.6 milioni di abitanti. Il programma del piano quinquennale che sarà concluso nel 1957, prevede un servizio su 472000 miglia quadrate con una popolazione di 220 milioni. Sicchè dei 357 milioni d'abitanti residenti su un'area di 1.3 milioni di miglia quadrate, le radiotrasmissioni serviranno circa il 60% della popolazione e il 37% dell'area totale.

Sempre nel 1947 vi erano 8 stazioni radio che collegavano l'India con i paesi stranieri per le comunicazioni radiotelegrafiche, radiotelefoniche e di radiofoto; nel Marzo 1956 vi sono ormai 41 stazioni che assicurano un collegamento con qualsiasi parte del globo.

I progressi accennati sono veramente sorprendenti per il notevole sforzo economico di cui si è dimostrato capace un paese ancora sottosviluppato. Inoltre occorre tener presente che tutto ciò è stato ottenuto senza disporre, in partenza, di una benchè minima industria elettronica nazionale e con una estrema carenza di personale tecnico. Tuttavia anche queste iniziali deficienze vanno mano a mano scomparendo: infatti a cura del governo indiano sono sorti diversi Istituti di Tecnologia e scuole per l'ingegneria elettronica, mentre sono incoraggiati con molto entusiasmo tutti i tentativi per lo sviluppo di una solida industria elettronica nazionale.

PELLEGRINI UMBERTO

Proceedings of the International Conference on the Peaceful Uses of Atomic Energy. Geneva, 8-20 August 1955, Vol. 7; Nuclear Chemistry and Effects of Irradiation, United Nations, New York, 1956.

La conferenza internazionale sull'uso pacifico dell'energia atomica tenutasi in Ginevra nell'Agosto del 1955 ha costituito un punto di incontro per gli scienziati di tutto il mondo che hanno avuto modo di scambiarsi notizie sullo stato degli studi e delle ricerche.

Sono stati comunicati a Ginevra dei dati che non erano prima ufficialmente noti o sui quali non era facile ottenere una esauriente documentazione. Non si deve credere d'altra parte che le relazioni presentate a Ginevra rappresentino una completa ed esauriente messa a punto di tutti i numerosi argomenti trattati: è evidente intanto che una parte delle conoscenze sino ad oggi acquistate si trova tuttora sotto il vincolo del segreto; le comunicazioni presentate a Ginevra sono poi molto spesso dei riassunti generali i quali non rispecchiano che indirettamente i contributi originali, e non sempre sono accompagnati da una adeguata documentazione. Ne consegue che, quando si tenti di sintetizzare e riassumere l'insieme di tutti i dati forniti su un determinato argomento vi si trovi qualche imperfezione e lacuna.

Nonostante tutto questo, i 16 volumi degli atti di questo grandioso convegno, pubblicati in quattro lingue a cura delle Nazioni Unite, costituiscono un poderoso complesso di informazioni nei campi più svariati.

Il settimo volume contiene le relazioni presentate sugli argomenti della Chimica nucleare e degli effetti delle radiazioni sui solidi e sui liquidi.

Il volume è diviso in dieci parti corrispondenti ad altrettante sessioni della conferenza, ciascuna delle quali è seguita dai resoconti delle sedute comprendenti le discussioni.

Nella prima parte quattro lavori sono dedicati ai processi di fissione osservati particolarmente dal punto di vista chimico; vi si trovano liste dei prodotti di fissione con la loro quantità percentuale misurata con i vari metodi, note sulla fissione spontanea, ecc.

L'argomento della seconda parte è di grande interesse pratico in quanto riguarda l'attrezzatura dei laboratori in cui si lavora con materiali altamente radioattivi: vengono descritti vari tipi di laboratorio, come quello di chimica analitica, quello metallografico, ecc. Presenta particolare interesse la possibilità di confronto tra i paesi maggiormente sviluppati in questo settore. La chimica delle sostanze radioattive viene poi trattata in cinque delle seguenti sezioni: procedimenti di lavorazione, ed in particolare di estrazione, applicati ai prodotti della pila; chimica dei prodotti di fissione, chimica degli elementi transuranici e pesanti e metodi di separazione.

Le ultime tre sezioni sono dedicate agli effetti della radiazione sulla materia: sui liquidi si tratta ancora prevalentemente di effetti chimici come le reazioni ridotte nelle sostanze organiche, la radiolisi dell'acqua, ecc.

Quanto viene discusso nelle due parti dedicate ai solidi ha invece con la chimica solo qualche punto di contatto, ma presenta il massimo interesse per gli studiosi di fisica dei solidi e per gli ingegneri che si preoccupano delle alterazioni delle proprietà dei materiali strutturali esposti all'intenso flusso dei neutroni ed altre radiazioni presenti nei reattori nucleari.

In effetto lo studio delle modificazioni del materiale strutturale dei reattori è stato forse, dal punto di vista storico, il primo movente delle ricerche in questo campo. Queste ricerche si sono poi sviluppate secondo due direttive principali: quella prevalentemente empirica e applicativa che si preoccupava principalmente dei nuovi sviluppi della tecnica dei materiali e quella più propriamente scienti-

fica che scopriva nello studio degli effetti delle radiazioni un potente mezzo di indagine sui difetti dei solidi cristallini e sulle loro relazioni con le proprietà fisiche dei solidi stessi. Questi studi che sono tuttora in pieno sviluppo e ben lungi da una definitiva sistemazione, hanno permesso un notevole progresso alla fisica dei solidi.

La suddivisione della materia nelle due sessoni della conferenza di Ginevra dedicata a questi argomenti rispecchia appunto questi due orientamenti. La sessione che tratta degli effetti sui materiali del reattore è formata principalmente da articoli riassuntivi ad opera di autori americani, inglesi e russi. Il quadro un po' schematico che ne risulta non può dirsi completo, specialmente per ciò che riguarda lo studio sui metalli nei quali i fenomeni sono di altissimo interesse ma, in condizioni di pratico impiego, le modificazioni delle proprietà macroscopiche difficilmente assumono valori preoccupanti dal punto di vista della ingegneria. Sembra invece di un certo rilievo l'interesse che viene dedicato alle modificazioni subite dai materiali fissionabili e dalla grafite, le quali assumono in certi casi aspetti assai vistosi ed assolutamente non trascurabili.

La parte del volume di maggiore interesse per quanto riguarda la fisica dei solidi è l'ultima: essa contiene due importanti articoli sugli aspetti teorici dei problemi del « Radiation Damage » ad opera di SEITZ l'uno e di DIENES l'altro. In questi articoli sono esposti alcuni dei più recenti contributi alla teoria.

Gli altri lavori forniscono un riassunto di risultati sperimentali e delle interpretazioni degli effetti prodotti dalle radiazioni specialmente su cristalli ionici di materiali non metallici; il problema degli effetti sulla grafite viene ripreso in questa sezione, in senso interpretativo, ad opera di G. R. HENNING e J. E. HOVE.

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